22S:30/105 Statistical Methods and Computing

Measures of Center, continued Measures of Dispersion

Lecture 3
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The mean is meaningful only for quantitative data (either discrete or continuous).

- Example regarding a discrete variable: We hear reports such as that the average number of children per family is 1.9.
- The mean is not meaningful for nominal or ordinal data.

Exception: if a binary variable is coded as 0 and 1.

Then the arithmetic mean is the proportion of observations in the dataset that have value 1.

Example:

- An ecological study of a habitat in which 10 rare species of bird are known to have lived as of 1990
- In 1999, a naturalist is sent to spend a day in the area and to record any members of these 10 species that she observes
- A variable is coded as follows:
  - 1 = at least one member of the species was observed
  - 0 = no members of the species were observed

species	observed?
1	1
2	0
3	1
4	1
5	1
6	0
7	1
8	1
9	1
10	1

- The mean

$$\bar{x} = \frac{8}{10} = .8$$

indicates that 80% of the species were observed.

The median

The median is the 50th percentile of a set of observations.

- Values must be sorted from smallest to largest.
- If the number of observations is odd, then the median is the middle value.

  75 80 82 88 95

  The median is 82.
- If the number of observations is even, then the usual way to define the median is as the **mean** of the **two** middle values.

  75 80 82 88 95 97

  The median is $\frac{82+88}{2} = 85$.

The median is **not** strongly affected by a few extreme values in the dataset.

Example 1:

75 80 82 88 95

- mean = 84
- median = 82

Example 2:

25 80 82 88 95

- mean = 74
- median = 82

The median is *robust* to extreme values.

The median can be used as a measure of center for **ordinal** data as well as for discrete and continuous data.

Example: The NYC poll

```
                                          Cumulative
city1yr    Frequency      Percent       Frequency
-----------------------------------------------
 Worse          593        61.64          593
 Same           252        26.20          845
 Better         111        11.54          956
```

- 956 people answered this question regarding whether they thought the condition of the city in June, 2003, was better, worse, or the same as one year earlier.
- If the values are sorted from smallest to largest (Worse, Same, Better), then the median will be the average of the 478th and 479th values.
- We can use the cumulative frequencies in the table to figure out what these have to be. They are both in category ”Worse.”
- Thus the median is Worse.

The mode

- The mode of a set of values is the value that occurs most frequently.
- Example: in the NYC poll data, the mode of the “city1yr” variable is Worse.
- Example: There is no mode in the birthweights data, because no value occurs more than once.
- There may be more than one mode in a set of values.
- The mode may be reported for **all** types of data.

When is each measure of central tendency appropriate?

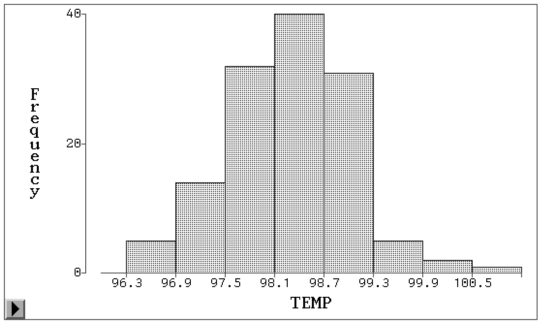
Depending on data type

- Nominal data
  - mode only
  - possible exception: binary data coded 0 and 1
- Ordinal data
  - mode or median
- Quantitative data
  - mean, median, or mode

Depending on the shape of the distribution of values (quantitative variables)

- if the shape is approximately symmetric and has only one mode
  - mean and median will be close in value
  - mean is typically reported

  Example: the body temperature data

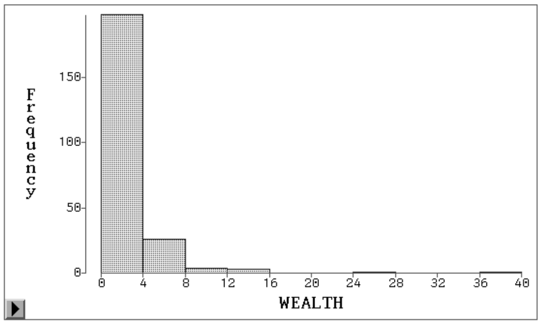


From a statistical computer package:

- mean = 98.24
- median = 98.3

- if the distribution is highly skewed
  - if skewed to the right, mean will be larger than median
  - if skewed to the left, mean will be smaller than median
  - mean may not be a "typical" value

  Example: the billionaire data

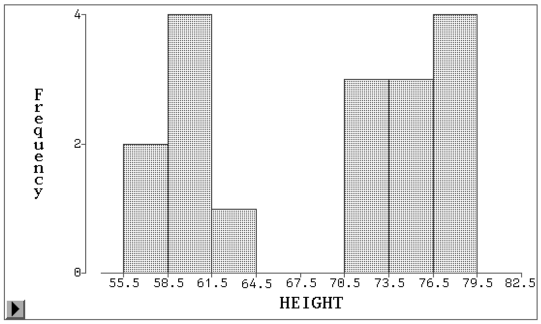


From a statistical computer package:

- mean = 2.7 billion
- median = 1.8 billion

- if the distribution has more than one mode
  - neither the mean nor the median may be representative values
  - may be best to report all modes and/or to display a graph
  - may occur if two or more different subgroups are represented in the sample

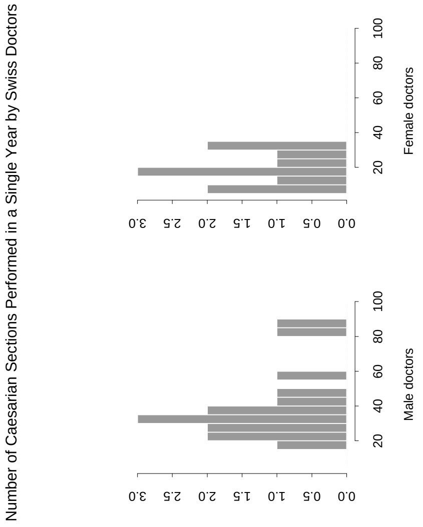
Example:



From a statistical computer package:

- mean = 69.0
- median = 72.0

In getting the "overall picture" of quantitative data, the spread is just as important as the center of the data.



Numerical measures of dispersion

- the range
- the interquartile range
- the standard deviation

The range

- The range is the difference between the largest and the smallest observations.
- For the male Swiss doctors,
  - largest value = 86
  - smallest value = 20
  - range = $86 - 20 = 66$
- For the female Swiss doctors,
  - largest value = 33
  - smallest value = 5
  - range = $33 - 5 = 28$

The range shows the full spread of the data, but may be exaggerated if the largest and/or smallest values are atypical (outliers)

- Example: the 1992 billionaire data
  - With Bill Gates:
    range = 37 - 1 = 36 billion
  - If Bill were deleted:
    range = 24 - 1 = 23 billion
- Example: the male Swiss doctors data
  - With the largest two values
    range = 86 - 20 = 66 billion
  - If the two largest values were deleted:
    range = 59 - 20 = 39 billion
- So additional measures are needed to give a more complete picture of the spread of values.

The quartiles and the interquartile range

- The *first quartile* is the same as the 25th percentile
  - one quarter of the observations in a dataset have values less than or equal to the 1st quartile, and the other three quarters have values greater than or equal to the first quartile
- The *third quartile* is the same as the 75th percentile
  - three quarters of the observations in a dataset have values less than or equal to the 3rd quartile, and the other one quarter have values greater than or equal to the 3rd quartile

- The interquartile range (IQR) is the difference between the 3rd and 1st quartiles
- For the male Swiss doctors,
  - third quartile = 50
  - first quartile = 27
  - IQR = 50 - 27 = 23
- For the female Swiss doctors,
  - third quartile = 29
  - first quartile = 14
  - IQR = $29 - 14 = 15$
- For the 1992 billionaires,
  - third quartile = 3 billion
  - first quartile = 1.3 billion
  - IQR = $3 - 1.3 = 1.7$ billion

The IQR is considered less sensitive to outliers than the range.

- Example: the 1992 billionaire data
  - With Bill Gates:
    IQR = 3 − 1.3 = 1.7 billion
  - If Bill were deleted:
    IQR = 2.9 − 1.3 = 1.6 billion

- However, in a small dataset, deletion of a few outliers may affect the IQR substantially.

- Example: the male Swiss doctors
  - IQR with the two largest values included:
  - IQR = 50 − 27 = 23
  - IQR with the two largest values deleted:
  - IQR = 37 − 27 = 10

The five-number summary

- The five-number summary provides a reasonably-complete numeric summary of the center and dispersion of a set of values.

- The five-number summary consists of
  - the minimum value
  - the first quartile
  - the median
  - the third quartile
  - the maximum value

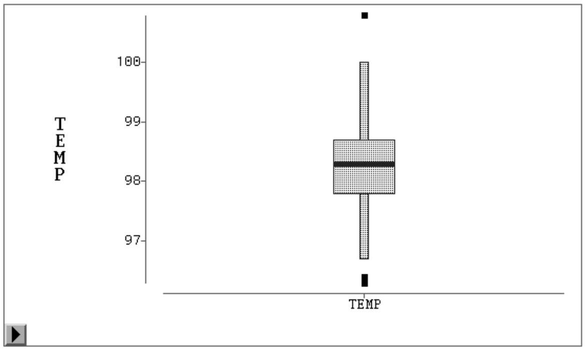
The five-number summary for the billionaire data may be extracted from the following computer output:

```
                Quantiles(Def=5)

     100% Max          37          99%          14
      75% Q3            3          95%         6.2
      50% Med         1.8          90%         4.5
      25% Q1          1.3          10%         1.1
       0% Min           1           5%           1
                                    1%           1
     Range             36
     Q3-Q1            1.7
     Mode               1
```

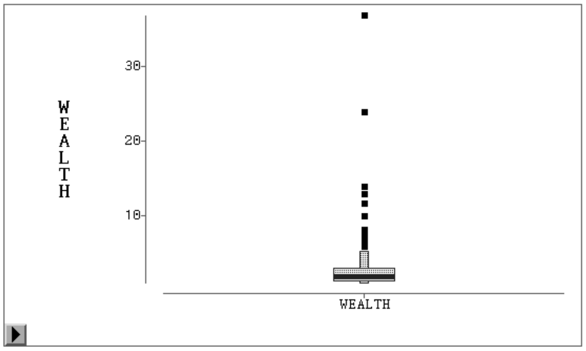
Boxplots

- are used to summarize the distribution of a continuous variable



- box extends from 1st quartile to 3rd quartile of data
- line in middle of box marks 50th percentile

- “whiskers” sticking out of box extend to *adjacent values*
  - adjacent values are most extreme observations that are not farther away from the edge of the box than 1.5 times the height of the box
- points farther out than the adjacent values are considered *outliers*
  - represented by circles or squares
  - probably are not typical of the rest of the data



The standard deviation

- The standard deviation measures spread by looking at how far the observations are from their mean.
- Example: quiz scores

  75 80 82 88 95

  The mean is

$$\begin{aligned}\bar{x} &= \frac{75+80+82+88+95}{5} \\ &= 84\end{aligned}$$

  points.
- We want a measure of typical distance between an individual value and this mean.

An idea that won’t work for measuring the spread: take the average of the “deviations” of the individual observations from the mean.

Observed Value	Deviation from mean		Squared deviation	
75	75 - 84 =	-9	$(-9)^2 =$	81
80	80 - 84 =	-4	$(-4)^2 =$	16
82	82 - 84 =	-2	$(-2)^2 =$	4
88	88 - 84 =	4	$4^2 =$	16
95	95 - 84 =	11	$11^2 =$	121
sum		0		238

Because the sum of the deviations is always 0, the average deviation is always 0!

Solution: Square the individual deviations before adding them up!

The variance and the standard deviation

- The variance s^2 is the sum of the squared deviations divided by one less than the number of observations.

$$\begin{aligned} s^2 &= \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \\ &= \frac{238}{4} = 59.5 \end{aligned}$$

  - We can think of the variance roughly as the average of the squared deviations.

- The standard deviation is the square root of the variance.
  $s = \sqrt{59.5} = 7.71$ points.

Facts about the standard deviation s

- s measures the spread of values around the *mean*
  - thus s should be used as a measure of dispersion only when mean has been chosen as the measure of center

- s is always greater than 0 unless all the observations have the same value

- s has same units of measurement as original observations

- s is sensitive to extreme observations
  - like the mean

- s is the most commonly-used measure of dispersion (is often used when it is not the best choice!)

The mean and standard deviation together provide a reasonable numeric summary of a set of values if the distribution is approximately **symmetric**.

- Example: the body temperature data

```
Variable    N          Mean       Std Dev
------------------------------------------
TEMP       130    98.2492308     0.7331832
```

- Example of inappropriate use of $\bar{x}$ and s to summarize a distribution: the billionaire data

```
 Analysis Variable : WLTH

  N          Mean       Std Dev
 -------------------------------
 233     2.6815451     3.3188403
 -------------------------------
```