## Homework

## Elementary Statistics \& Inference (STAT:1020; Bognar)

1. Let $p$ denote the population proportion of (budget) LCD panels that have defective pixels. A researcher randomly selected 100 LCD panels and determined that 8 have defective pixels.
(a) Suppose we wish to test $H_{0}: p=0.12$ versus $H_{a}: p \neq 0.12$ at the $\alpha=0.05$ significance level using the score test. Approximate the $p$-value for this test.
(b) In reference to 1a, does the proportion of defective panels significantly differ from 0.12 ? Why?
(c) Find an approximate $95 \%$ Wald confidence interval for $p$.
(d) Based upon your answer in 1 C , does the proportion of defective panels significantly differ from 0.12 ? Why?
(e) What is the estimated standard error of $\hat{p}, \widehat{s e}(\hat{p})$ ?
(f) What is the estimated margin of error (at $95 \%$ confidence)?
(g) How many panels would need to be selected for the margin of error (at $95 \%$ confidence) to equal 0.03 ?
2. Suppose the population proportion of computer keyboards that fail is $p$. To infer about $p$, a manufacturer randomly selected 60 keyboards and determined that 6 had failed.
(a) Find an approximate $95 \%$ Wald confidence interval for $p$.
(b) Find an approximate $95 \%$ Agresti-Coull confidence interval for $p$.
(c) Based upon your answer in 2b, does $p$ significantly differ from 0.15 ? Why?
(d) Based upon your answer in 2 b , does $p$ significantly differ from 0.50 ? Why?
(e) Suppose we wish to test $H_{0}: p=0.50$ versus $H_{a}: p \neq 0.50$ at the $\alpha=0.05$ significance level. Based upon your answer in 2 b will the $p$-value for the test be less than or greater than $\alpha$ ? Why?
3. A retailer is seeking to study the satisfaction of its customers. The population proportion of satisfied customers is $p$. A random sample of 100 customers yielded 90 that were satisfied. We would like to determine if more than $80 \%$ of the customers are satisfied.
(a) Test $H_{0}: p=0.8$ versus $H_{a}: p>0.8$ at the $\alpha=0.05$ significance level using the score test. Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.
(b) Find the $p$-value for the test in 3 a
(c) Based upon your answer in 3b, is the proportion of satisfied customers significantly higher than 0.80 ? Why?
4. After smoking marijuana, 7 out of 100 subjects failed a driving test on the Iowa Driving Simulator. Only 7 out of 140 subjects not under the influence of marijuana failed the test. Let $p_{1}$ denote the population proportion of marijuana users that fail the test, and let $p_{2}$ denote the population proportion of drivers not under the influence of marijuana that fail the driving test.
(a) Find an $80 \%$ confidence interval for $p_{1}-p_{2}$. Is there a significant difference between the groups? Why?
(b) Suppose we wish to test $H_{0}: p_{1}=p_{2}$ versus $H_{a}: p_{1} \neq p_{2}$. Find the $p$-value for this test. Is there a significant difference in the proportion of marijuana users and non-marijuana users fail the test? Why?
5. A high-quality diamond cutter can receive raw diamonds from two mines: one is in Russia and the other is in Africa. The Russian mine charges slightly more for its diamonds. However, if the Russian mine has a higher proportion of "colorless" diamonds, the diamond cutter will choose to buy his diamonds from them. Let $p_{1}$ denote the proportion of colorless diamonds from the Russian mine, and let $p_{2}$ denote the proportion of colorless diamonds from the African mine. A random sample of 50 diamonds from each mine found the following:

$$
\begin{aligned}
\text { Russia: } & n_{1}=50, \text { number of colorless }=10 \\
\text { Africa: } & n_{2}=50, \text { number of colorless }=6
\end{aligned}
$$

(a) Test $H_{0}: p_{1}=p_{2}$ versus $H_{a}: p_{1}>p_{2}$ at the $\alpha=0.05$ significance level. Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.
(b) Find the $p$-value for the test in 5 a.
(c) Based upon your analysis, is it worthwhile for the diamond cutter to pay extra for diamonds from the Russian mine? Why?

