Homework
Elementary Statistics & Inference (STAT:1020; Bognar)

1. A bowl contains 3 chips: the chips are labeled 0, 2, and 4. A chip is randomly selected from the bowl. Let X denote the number printed on the chip. The probability mass function (probability distribution) of X is

x :	0	2	4
$P(X = x)$:	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

   (a) Find the mean of X, $\mu = E(X)$.

   (b) Find the standard deviation of X, $\sigma = SD(X)$.

   (c) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find the sampling distribution of $\bar{X}$ using the method shown in lecture. Write your answer in tabular form, i.e.

$$\begin{array}{r} \bar{x} : \\ P(\bar{X} = \bar{x}) : \end{array}$$

   Hint: Write out all 9 outcomes in the sample space, find $\bar{x}$ for each outcome, then find the sampling distribution of $\bar{X}$. All 9 outcomes in the sample space are equally likely.

   (d) Determine the mean of $\bar{X}$ using $\mu_{\bar{X}} = E(\bar{X}) = \sum_{\bar{x}} \bar{x} P(\bar{X} = \bar{x})$.

   (e) Determine the standard deviation of $\bar{X}$ using $\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{\sum_{\bar{x}} \left(\bar{x} - E(\bar{X})\right)^2 P(\bar{X} = \bar{x})}$.

2. Suppose a bottling plant fills 2-liter soda bottles. The distribution of the amount of soda dispensed into each bottle follows a normal distribution with mean $\mu = 2.02$ liters and standard deviation $\sigma = 0.009$ liters.

   (a) Find the probability that a randomly selected bottle contains more than 2.03 liters.

   (b) Find the probability that the mean amount of soda $\bar{X}$ in 36 randomly selected bottles is greater than 2.022 liters.

   (c) Find the probability that the mean amount of soda $\bar{X}$ in 4 randomly selected bottles is between than 2.010 and 2.015 liters.

   (d) Suppose 4 bottles of soda are randomly selected. Determine the 80th percentile of the sample mean $\bar{X}$.

3. Based upon past data, a professor knows that the number of absent students on any given day is strongly skewed to the right with mean $\mu = 8$ and standard deviation $\sigma = 12$.

   (a) Suppose 4 days are randomly selected (assume independence). Can you find the probability that the mean number of absences, $\bar{X}$, is less than 2? If so, find the probability. If not, explain why.

   (b) Suppose the class meets 32 times during the semester (assume independence). Approximate the probability that the mean number of daily absences during the semester, $\bar{X}$, is less than 7.

4. The expenditures (in dollars) of customers at a coffee shop has a distribution that is strongly skewed to the right with mean $\mu = 3.50$ and standard deviation $\sigma = 2.00$.

   (a) Suppose 12 customers enter the shop (assume independence). Can you find the probability that the mean expenditure, $\bar{X}$, is more than \$3.75? If so, find the probability. If not, explain why.

   (b) Suppose 100 customers are randomly selected (assume independence). Approximate the probability that the mean expenditure, $\bar{X}$, is more than \$3.00.

   (c) Suppose 100 customers are randomly selected (assume independence). Approximate the probability that the mean expenditure, $\bar{X}$, is between than \$3.00 and \$3.25.

   (d) Suppose 100 customers are randomly selected (assume independence). Find the 99th percentile of the sample mean expenditure $\bar{X}$.