Homework 4
Prob. and Stat. for Eng. (STAT:2020; Bognar)

1. Suppose the random variable X has the following probability distribution. Find the following probabilities.

x:	0	1	2	3	4
$f(x) = P(X = x)$:	0.2	0.1	0.2	0.2	0.3

   (a) $P(X \leq 2)$
   (b) $P(X < 2)$
   (c) $P(X \leq 2 \cap X \geq 1)$
   (d) $P(X = 1 \cup X < 3)$
   (e) $P(X = 2|X \leq 2)$

2. A large warehouse contains 2-packs, 4-packs, and 8-packs of batteries. Suppose the random variable X equals the number of batteries in a randomly selected package of batteries. It is known that X has probability distribution

$$f(x) = P(X = x) = \frac{8}{7x} \quad \text{for } x = 2, 4, 8$$

   (a) What is $P(X = 2)$?
   (b) Determine $P(X \geq 4)$.

3. Suppose the discrete random variable X has probability distribution

$$f(x) = P(X = x) = \frac{1}{2^x} \quad \text{for } x = 1, 2, \ldots$$

   (a) Find $P(X = 5)$.
   (b) Determine $P(X \geq 2)$.
   (c) Find $P(X \leq 4 \cap X \geq 4)$.
   (d) Find $P(X \leq 4 \cup X \geq 4)$.
   (e) Determine $P(X \leq 3|X \geq 2)$.

4. A basket contains 4 puppies: one of the puppies has 1 spot, one of the puppies has 2 spots, and the remaining two puppies have 4 spots. Suppose *two* puppies are selected at random *without* replacement. Let the random variable X equal the *total* number of spots on the selected puppies.

   (a) Find the probability distribution of X.
   (b) Find the probability that the puppies have a total of 5 spots, i.e. find $P(X = 5)$.
   (c) Find the probability that the puppies have a total of 6 or more spots, i.e. find $P(X \geq 6)$.

5. Suppose a bowl has 9 chips; one chip is labeled "1", three chips are labeled "3", and five chips are labeled "5". Suppose *two* chips are selected at random *with* replacement. Let the random variable X equal the *absolute difference* between the two draws (e.g. if the first draw is a 1 (1_1) and the second draw is a 5 (5_2), then the absolute difference is $|1 - 5| = 4$).

   (a) Find the probability distribution of X.
   (b) Find the probability that both draws are the same.
   (c) Find the probability that both draws are *not* the same.
   (d) Given that both draws are *not* the same, determine the probability that the absolute difference is equal to 2, i.e. find $P(X = 2|X > 0)$.

6. Repeat question 5(a) assuming the chips are drawn *without* replacement.