

HOMEWORK (BOGNAR)
INTRODUCTION TO MATHEMATICAL STATISTICS II (STAT:3101)

Using the Hogg, Tanis, Zimmerman parameterization, if $Y \sim \text{exp}(\theta)$, then $f_Y(y) = \frac{1}{\theta}e^{-y/\theta}$ for $y > 0$, $E(Y) = \theta$, and $\text{Var}(Y) = \theta^2$.

1. Suppose the conditional pdf of X given Y is $f_{X|Y}(x) = 2x/Y^2$ for $0 < x < Y$ where $Y \sim \text{Unif}(0, 1)$.
 - (a) Find $E(X|Y)$ (your answer will be a function of Y).
 - (b) Find $\text{Var}(X|Y)$ (your answer will be a function of Y).
 - (c) Using the law of total expectation, find $E(X)$.
 - (d) Using the law of total variance, find $\text{Var}(X)$.

2. Suppose $X|Y \sim \text{Unif}(0, Y^2)$ where Y has pdf $f_Y(y) = 4y^3$ for $0 < y < 1$.
 - (a) Find the joint pdf of X and Y , $f_{XY}(x, y)$. *Be sure to state the joint support.*
 - (b) Find the marginal pdf of X , $f_X(x)$. *Be sure to state the support.*
 - (c) Using the marginal pdf of X , find $E(X)$.
 - (d) Using the marginal pdf of X , find $\text{Var}(X)$.
 - (e) Find $E(X|Y)$ (your answer will be a function of Y).
 - (f) Find $\text{Var}(X|Y)$ (your answer will be a function of Y).
 - (g) Using the law of total expectation, find $E(X)$.
 - (h) Using the law of total variance, find $\text{Var}(X)$.