Homework (Bognar)
Introduction to Mathematical Statistics II (STAT:3101)

1. The standard Gumbel distribution has support $\mathcal{S}_X = (-\infty, \infty)$ with pdf

$$f_X(x) = e^{-(x+e^{-x})}$$

and cdf

$$F_X(x) = P(X \leq x) = e^{-e^{-x}}$$

Five random numbers $y_1, \ldots, y_5$ were generated from a $Unif(0,1)$ distribution. Using these random numbers, generate a random sample $x_1, \ldots, x_5$ from the standard Gumbel distribution.

$$\begin{array}{ll} y_1 = 0.924 & x_1 = \\ y_2 = 0.538 & x_2 = \\ y_3 = 0.007 & x_3 = \\ y_4 = 0.358 & x_4 = \\ y_5 = 0.805 & x_5 = \end{array}$$

2. Suppose X_1 and X_2 have joint pdf

$$f_{X_1X_2}(x_1, x_2) = 24x_1x_2$$

for $0 < x_1 < 1$ and $0 < x_2 < 1 - x_1$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_2$.

   (a) Find the joint pdf of Y_1 and Y_2, $f_{Y_1Y_2}(y_1, y_2)$. *Be sure to state the joint support.*

   (b) Find the marginal pdf of Y_1, $f_{Y_1}(y_1)$. *Be sure to state the support. Compare this marginal pdf to the result from lecture; are you surprised that the marginals match?*

   (c) Find $Var(X_1 + X_2) = Var(Y)$.

3. Suppose X_1 and X_2 have joint pdf

$$f_{X_1X_2}(x_1, x_2) = 2$$

for $0 < x_1 < x_2 < 1$. Let $Y_1 = X_2/X_1$ and $Y_2 = X_1$.

   (a) Find the joint pdf of Y_1 and Y_2, $f_{Y_1Y_2}(y_1, y_2)$. *Be sure to state the joint support.*

   (b) Find the marginal pdf of Y_1, $f_{Y_1}(y_1)$. *Be sure to state the support.*