8.1 Sampling distributions

- What we've already covered from 8.1…
  - Distribution of the sample mean $\bar{X}$
- What we're covering now in 8.1…
  - Distribution of the sample proportion $\hat{p}$

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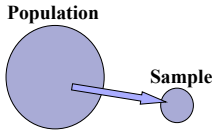
Estimating the population proportion p using the sample proportion $\hat{p}$

- Recall, we often want to make a statement about the population based on a random sample taken from a population of interest.



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Population Parameter	Sample Statistic
Population mean μ	Sample mean $\bar{X}$
The mean house value for all houses in Iowa	The mean house value for a sample of n=200 houses in Iowa
Population proportion p	Sample proportion $\hat{p}$
The proportion of all houses in Iowa with lead paint.	The proportion of Iowa houses in a sample of n=200 with lead paint.

↑ Unknown, but estimated from → ↑ Calculated from sample

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Population proportion p

- The parameter of interest when each subject (or observation) expresses one of two things…
  - In favor of proposition? Yes or no.
  - Alive at age 100? Yes or no.
  - Piece of candy: Orange or Not orange?

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Population proportion p

- Population
  - The population proportion p of all university of Iowa students who have exactly one sibling.
- Sample of 100 Iowa students
  - The proportion of the sample who have exactly one sibling.

$$\hat{p} = \frac{?}{100}$$

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Sample-to-sample variability

- All the general concepts we learned about estimating μ using $\bar{x}$ will apply when we estimate p using the sample proportion $\hat{p}$.
- The **sampling error** is the error introduced because a random sample is used to estimate a population parameter… also applies to proportions.

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What did we learn with $\bar{X}$?

- Different samples will produce different sample proportions $\hat{p}$.
- There is variation from one sample proportion to the next.
- Can we *model* this variation?
  - i.e. Does the $\hat{p}$ statistic vary in a *predictable* way?

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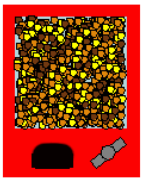
Simulation



- Population
  - Reeses Pieces
- Population Parameter
  - Proportion of Orange Reeses Pieces

A particular sample of size n = 10.

www.rossmanchance.com/applets/Reeses/ReesesPieces.html

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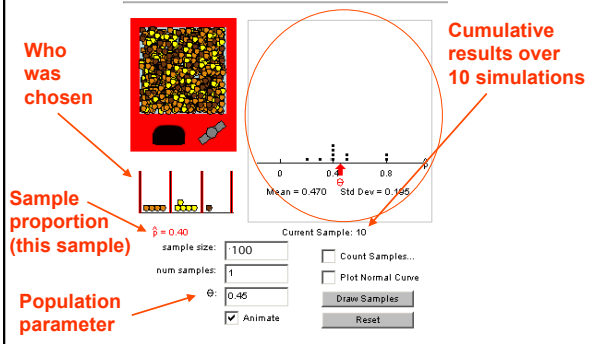
Reese's Pieces Samples



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Simulation

- Simple random sample of size n=100.
- Repeat several times.
- Record the sample proportion of orange Reeses Pieces.

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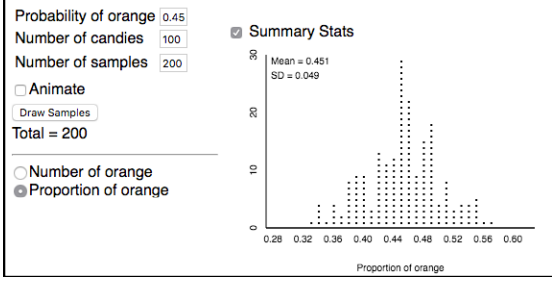
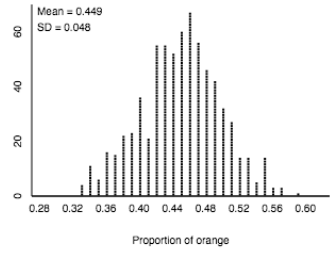
Simulation repeated 200 times

This histogram is a summary of the 200 estimates we got from 200 different samples of size 100.



Simulation repeated 500 times



This histogram is a summary of the 500 estimates we got from 500 different samples of size 100.

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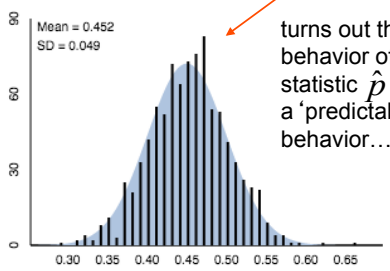
Simulation repeated 900 times



In the long run…

turns out that the behavior of our sample statistic $\hat{p}$ does have a 'predictable' behavior…

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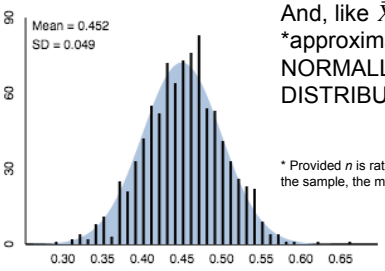
Simulation repeated 900 times



And, like $\bar{X}$, it's also *approximately NORMALLY DISTRIBUTED!!

* Provided *n* is rather large. And the larger the sample, the more normal it is.

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Sampling Distribution of $\hat{p}$

- Shape: Approximately Normal
- Center: The mean is p.
- Spread: The standard deviation is

$$\sqrt{\frac{p(1-p)}{n}}$$

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Exercise 1 (problem 8.1 #18):

- Suppose you know that the distribution of sample proportions of women employees is normal with a mean of p=0.42 and a standard deviation of 0.21. We take a sample and find the sample proportion $\hat{p} = 0.45$.
  - A) How many standard deviations is the sample proportion from the mean of the distribution of all sample proportions?

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- A)

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Exercise 1 (problem 8.1 #18):

- Suppose you know that the distribution of sample proportions of women employees is normal with a mean of p=0.42 and a standard deviation of 0.21. We take a sample and find the sample proportion $\hat{p} = 0.45$.
  - B) What is the probability that a second sample selected would have a proportion greater than 0.45?

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- B)

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Exercise 2:

- Suppose the proportion of the population in favor of a new law is 0.60. If you take a sample of size n=500, what is the probability that your sample proportion is 0.55 or lower?

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- Answer

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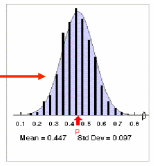
Note on Sampling Distributions

- Usually, we will only take one sample of a population and compute one statistic (e.g. $\hat{p}$).
  - You get one shot at estimating your parameter p !
- A Sampling Distribution gives us an idea of what we expect to get for our <u>one</u> statistic value.

If we drew an infinite number of random samples and calculated the sample statistic $\hat{p}$ for each, the sampling distribution would look like the resulting histogram of all these $\hat{p}$ values.



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