9.2 **Critical Values** for Statistical Significance in Hypothesis testing

Step 3 of Hypothesis Testing

- Step 3 involves computing a probability, and for this class, that means using the normal distribution and the z-table in Appendix A.

- What normal distribution will we use?
  - For p ?
  - For μ ?

Step 3:

- What normal distribution?
  - For a hypothesis test about μ, we will use…

We plug-in s here as our estimate for σ.

$$\bar{X} \sim N(\mu_{\bar{x}} = \mu_0, \sigma_{\bar{x}} = \sigma / \sqrt{n})$$

We assume the null is true, so we put the stated value of μ from the null hypothesis here.

Step 3:

- What normal distribution?
  - For a hypothesis test about p, we will use…

$$\hat{p} \sim N\left(p_0, \sqrt{\frac{p_0(1-p_0)}{n}}\right)$$

We assume the null is true, so we put the stated value of p from the null hypothesis into the formula for the mean and standard deviation.

Book example (Section 9.2, p.380):

- The null and alternative hypotheses are

$$H_0 : \mu = \$39{,}000$$

$$H_a : \mu < \$39{,}000 \qquad \text{(one-sided test)}$$

Data summary:

n=100 $\quad \bar{x} = \$37{,}000 \quad$ *s*=$6,150

Test of Hypothesis for μ

- Step 3: What normal distribution?

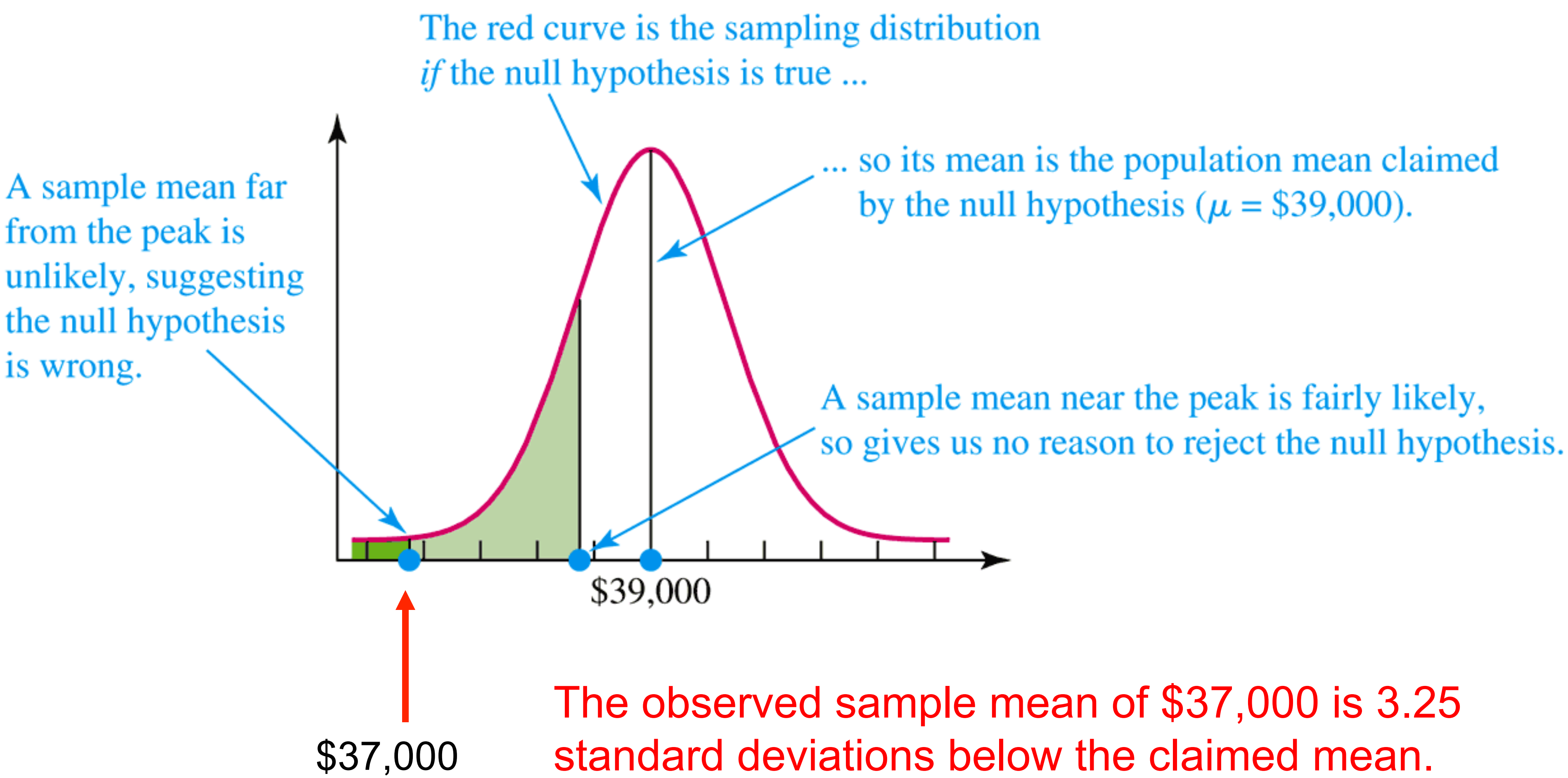
$$\bar{X} \sim N(\mu_{\bar{x}} = \mu_0, \sigma_{\bar{x}} = \sigma / \sqrt{n})$$

null hypothesis assumed true

$$\bar{X} \sim N(\mu_{\bar{x}} = \$39{,}000, \sigma_{\bar{x}} = \$6{,}150 / \sqrt{100})$$

From this normal distribution we can compute a z-score for our $\bar{x} = \$37{,}000$:

$$z = \frac{37{,}000 - 39{,}000}{6{,}150 / \sqrt{100}} = -3.25$$



The observed sample mean of \$37,000 is 3.25 standard deviations below the claimed mean.

What z-score could I get that will make me reject $H_0: \mu = \mu_0$?

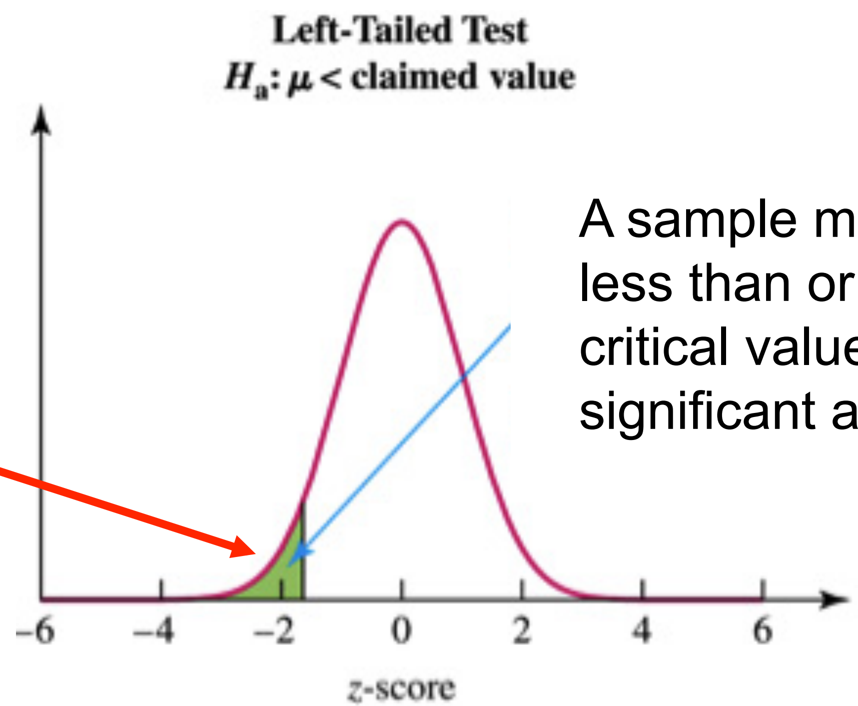
- It would have to be something in the ‘tail’ of the z-distribution (i.e. something far from the assumed true mean μ_0).
- It would have to suggest that my observed data is unlikely to occur under the null being true (small P-value).
- What about z=4? What about z=2?

Critical Values for Statistical Significance

- The z-score needed to reject H_0 is called the **critical value** for significance.
- The **critical value** depends on the significance level, which we state as α.
- Each type of alternative hypothesis has it's own critical values:
  - One-sided left-tailed test
  - One-sided right-tailed test
  - Two-sided test

Critical Values for Statistical Significance

- Significance level of **0.05**
  - One-sided left-tailed test $H_a: \mu < \mu_0$
    - Critical value is $z = -1.645$

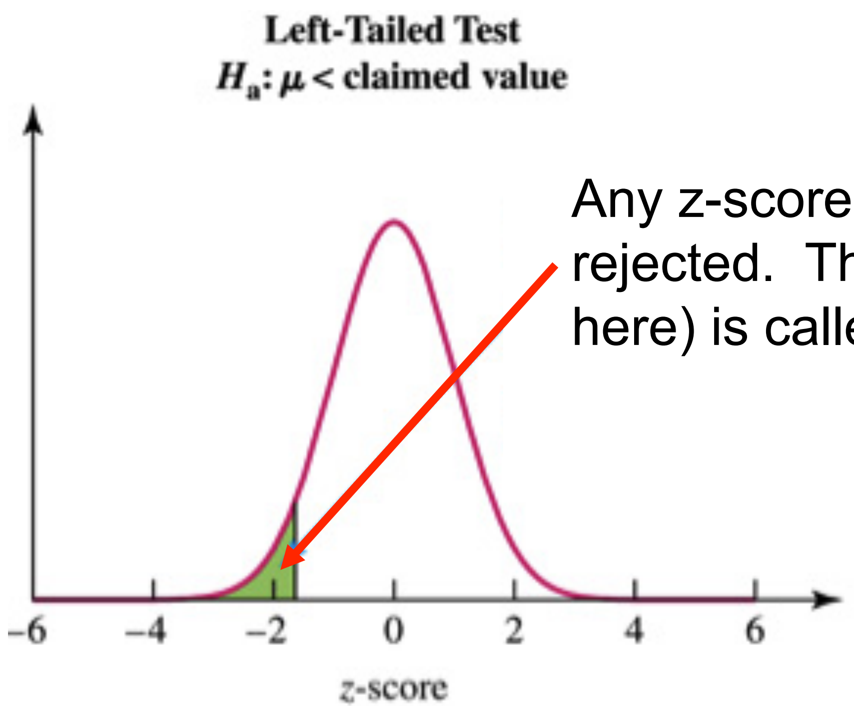


There is 0.05 to the left of the critical value.

A sample mean with a z-score less than or equal to the critical value of -1.645 is significant at the 0.05 level.

Critical Values for Statistical Significance

- Significance level of **0.05**
  - One-sided left-tailed test H_a: $\mu < \mu_0$
    - Critical value is $z = -1.645$



Any z-score to the left of -1.645 will be rejected. This zone (shown in green here) is called the **Rejection Region**.

If your z-score falls in the Rejection Region, you will reject the null.

Critical Values for Statistical Significance

- Significance level of **0.05**
  - One-sided left-tailed test $H_a: \mu < \mu_0$
    - Critical value is $z = -1.645$

- Book example:

$$H_0: \mu = \$39{,}000$$

$$H_a: \mu < \$39{,}000 \qquad \text{(one-sided test)}$$

$$z = \frac{37{,}000 - 39{,}000}{6{,}150 / \sqrt{100}} = -3.25$$

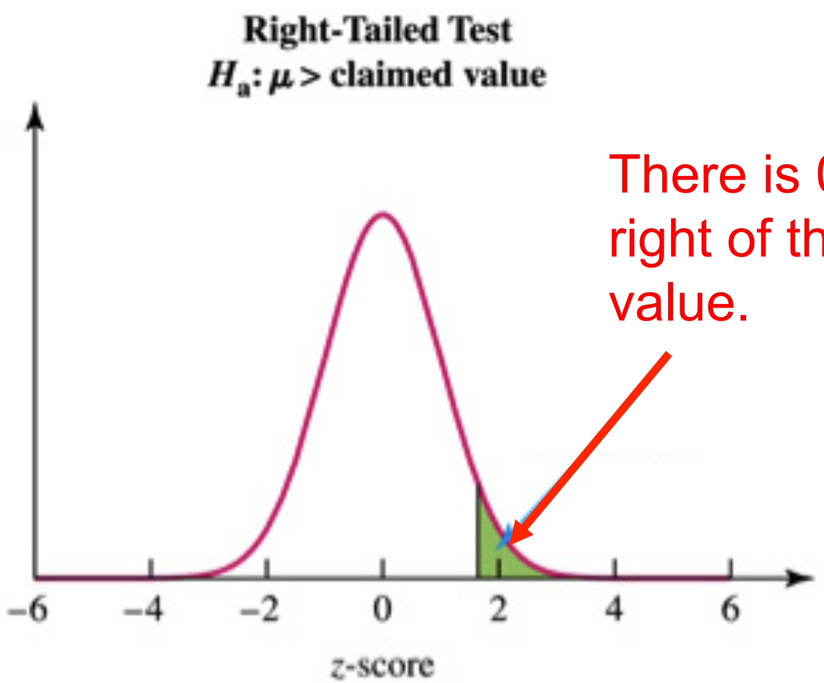
DECISION: The sample mean has a z-score less than or equal to the critical value of -1.645. Thus, it is significant at the 0.05 level.

z = -3.25 falls in the **Rejection Region**.

Critical Values for Statistical Significance

- Significance level of **0.05**
  - One-sided right-tailed test $H_a: \mu > \mu_0$
    - Critical value is $z = 1.645$

A sample mean with a z-score greater than or equal to the critical value of 1.645 is significant at the 0.05 level.



There is 0.05 to the right of the critical value.

Critical Values for Statistical Significance

- Significance level of **0.05**
  - One-sided right-tailed test $H_a: \mu > \mu_0$
    - Critical value is $z = 1.645$

- iTunes library example:

$$H_0: \mu = 7000$$

$$H_a: \mu > 7000 \qquad \text{(one-sided test)}$$

$$z = \frac{7160 - 7000}{1200/\sqrt{250}} = 2.11$$

DECISION: The sample mean has a z-score greater than or equal to the critical value of 1.645. Thus, it is significant at the 0.05 level.

z = 2.11 falls in the **Rejection Region**.

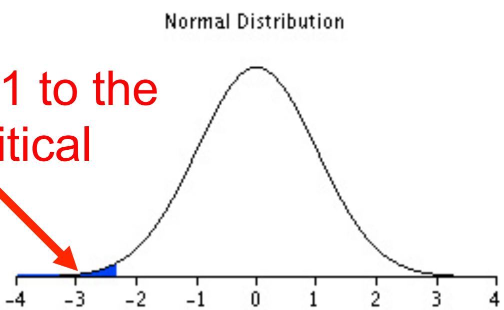
Critical Values for Statistical Significance

- Significance level of **0.01**
  - The same concept applies, but the critical values are farther from the mean.

H_0: $\mu = \mu_0$

H_a: $\mu < \mu_0$ (one-sided test)

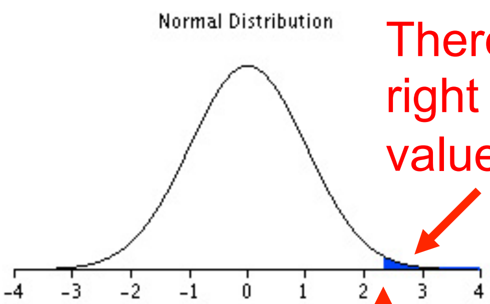
There is 0.01 to the left of the critical value.



$z = -2.33$

H_0: $\mu = \mu_0$

H_a: $\mu > \mu_0$ (one-sided test)



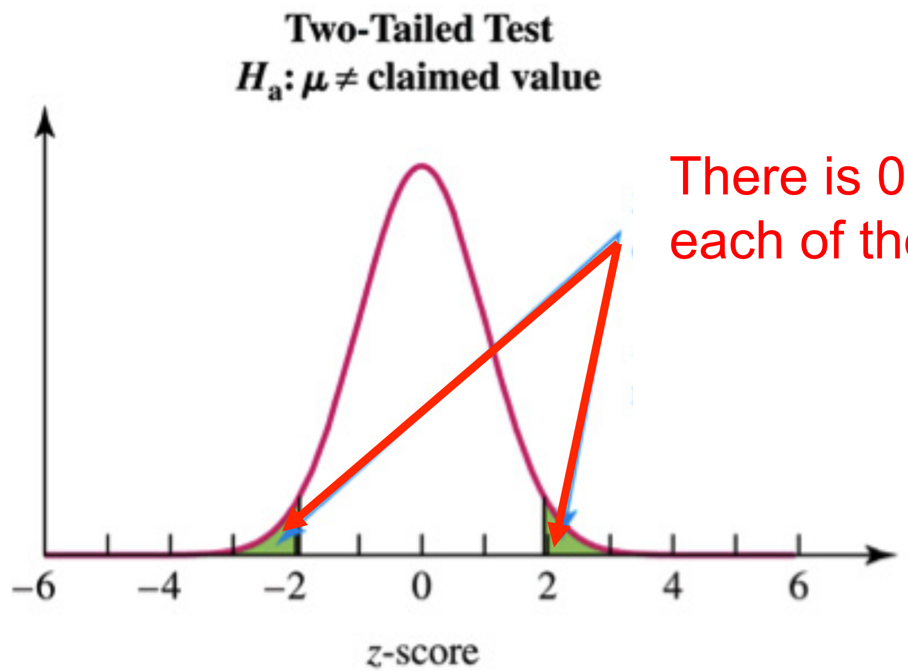
There is 0.01 to the right of the critical value.

$z = 2.33$

Critical Values for Statistical Significance

- Significance level of **0.05**
  - Two-sided test $H_a: \mu \neq \mu_0$ (two critical values)
    - Critical values are $z = -1.96 \ and \ z = 1.96$

A sample mean with a z-score in the rejection region (shown in green) is significant at the 0.05 level.



There is 0.025 in each of the tails.

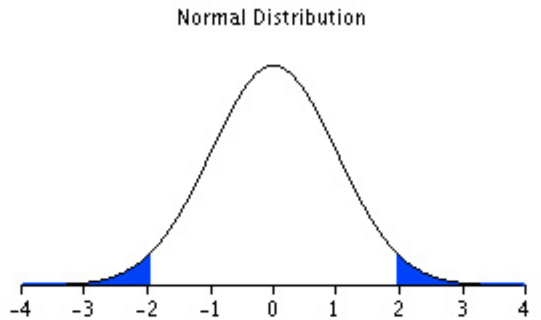
Critical Values for Statistical Significance

- Significance level of **0.05**
  - Two-sided test $H_a: \mu \neq \mu_0$ (two critical values)
    - Critical values are $z = -1.96 \quad and \quad z = 1.96$

- Spindle diameter example: $H_0: \mu = 5\text{mm}$

$H_a: \mu \neq 5\text{mm} \quad$ (two-sided test)

$$z = \frac{5.16 - 5}{1.56 / \sqrt{100}} = 1.02$$



DECISION: The sample mean has a z-score that is NOT in the 0.05 rejection region (shown in blue). Thus, it is NOT significant at the 0.05 level.

z = 1.02 does NOT fall in the **Rejection Region**.