

9.2 Critical Values for Statistical Significance in Hypothesis testing

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Step 3 of Hypothesis Testing

- Step 3 involves computing a probability, and for this class, that means using the normal distribution and the z-table in Appendix A.
- What normal distribution will we use?
 - For p ?
 - For μ ?

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Step 3:

- What normal distribution?
 - For a hypothesis test about μ , we will use...

We plug-in s here as our estimate for σ .

$$\bar{X} \sim N(\mu_{\bar{x}} = \mu_0, \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}})$$

We assume the null is true, so we put the stated value of μ from the null hypothesis here.

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Step 3:

- What normal distribution?
 - For a hypothesis test about p , we will use...

$$\hat{p} \sim N\left(p_0, \sqrt{\frac{p_0(1-p_0)}{n}}\right)$$

↑ ↑
 We assume the null is true, so we put the stated value of p from the null hypothesis into the formula for the mean and standard deviation.

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Book example (Section 9.2, p.380):

- The null and alternative hypotheses are

$$H_0: \mu = \$39,000$$

$$H_a: \mu < \$39,000 \quad (\text{one-sided test})$$

Data summary:

$n=100 \quad \bar{x} = \$37,000 \quad s = \$6,150$

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Test of Hypothesis for μ

- Step 3: What normal distribution?

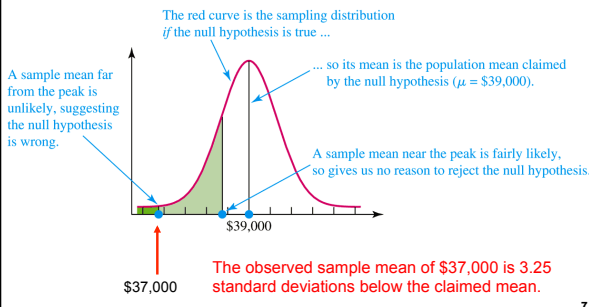
$$\bar{X} \sim N(\mu_{\bar{x}} = \mu_0, \sigma_{\bar{x}} = \sigma / \sqrt{n})$$

null hypothesis assumed true /
↑

$$\bar{X} \sim N(\mu_{\bar{x}} = \$39,000, \sigma_{\bar{x}} = \$6,150 / \sqrt{100})$$

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From this normal distribution we can compute a z-score for our $\bar{x} = \$37,000$:

$$z = \frac{37,000 - 39,000}{6,150 / \sqrt{100}} = -3.25$$


The red curve is the sampling distribution if the null hypothesis is true ...

... so its mean is the population mean claimed by the null hypothesis ($\mu = \$39,000$).

A sample mean far from the peak is unlikely, suggesting the null hypothesis is wrong.

A sample mean near the peak is fairly likely, so gives us no reason to reject the null hypothesis.

The observed sample mean of \$37,000 is 3.25 standard deviations below the claimed mean.

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What z-score could I get that will make me reject $H_0: \mu = \mu_0$?

- It would have to be something in the 'tail' of the z-distribution (i.e. something far from the assumed true mean μ_0).
- It would have to suggest that my observed data is unlikely to occur under the null being true (small P-value).
- What about $z=4$? What about $z=2$?

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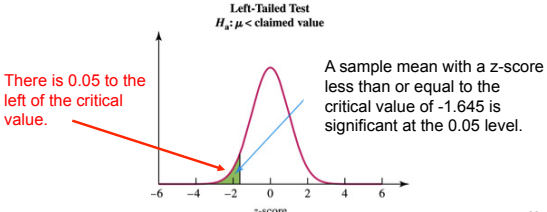
Critical Values for Statistical Significance

- The z-score needed to reject H_0 is called the **critical value** for significance.
- The **critical value** depends on the significance level, which we state as α .
- Each type of alternative hypothesis has it's own critical values:
 - One-sided left-tailed test
 - One-sided right-tailed test
 - Two-sided test

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - One-sided **left-tailed** test $H_a: \mu < \mu_0$
 - Critical value is $z = -1.645$



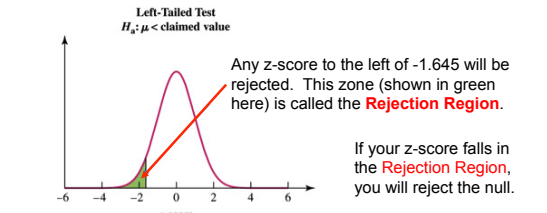
There is 0.05 to the left of the critical value.

A sample mean with a z-score less than or equal to the critical value of -1.645 is significant at the 0.05 level.

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - One-sided **left-tailed** test $H_a: \mu < \mu_0$
 - Critical value is $z = -1.645$



Any z-score to the left of -1.645 will be rejected. This zone (shown in green here) is called the **Rejection Region**.

If your z-score falls in the **Rejection Region**, you will reject the null.

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - One-sided **left-tailed** test $H_a: \mu < \mu_0$
 - Critical value is $z = -1.645$

■ Book example: $H_0: \mu = \$39,000$
 $H_a: \mu < \$39,000$ (one-sided test)

$$z = \frac{37,000 - 39,000}{6,150 / \sqrt{100}} = -3.25$$

DECISION: The sample mean has a z-score less than or equal to the critical value of -1.645. Thus, it is significant at the 0.05 level.

$z = -3.25$ falls in the **Rejection Region**.

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - One-sided **right-tailed** test $H_a: \mu > \mu_0$
 - Critical value is $z = 1.645$

A sample mean with a z-score greater than or equal to the critical value of 1.645 is significant at the 0.05 level.

z-score

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - One-sided **right-tailed** test $H_a: \mu > \mu_0$
 - Critical value is $z = 1.645$

- iTunes library example: $H_0: \mu = 7000$
 $H_a: \mu > 7000$ (one-sided test)

$$z = \frac{7160 - 7000}{1200 / \sqrt{250}} = 2.11$$

DECISION: The sample mean has a z-score greater than or equal to the critical value of 1.645. Thus, it is significant at the 0.05 level.

$z = 2.11$ falls in the **Rejection Region**.

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Critical Values for Statistical Significance

- Significance level of **0.01**
 - The same concept applies, but the critical values are farther from the mean.

$H_0: \mu = \mu_0$ $H_a: \mu < \mu_0$ (one-sided test)	$H_0: \mu = \mu_0$ $H_a: \mu > \mu_0$ (one-sided test)
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z = -2.33

z = 2.33

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - **Two-sided** test $H_a: \mu \neq \mu_0$ (**two** critical values)
 - Critical values are $z = -1.96$ and $z = 1.96$

A sample mean with a z-score in the rejection region (shown in green) is significant at the 0.05 level.

Two-Tailed Test
 $H_a: \mu \neq \text{claimed value}$

There is 0.025 in each of the tails.

z-score

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Critical Values for Statistical Significance

- Significance level of **0.05**
 - **Two-sided** test $H_a: \mu \neq \mu_0$ (**two** critical values)
 - Critical values are $z = -1.96$ and $z = 1.96$

- Spindle diameter example: $H_0: \mu = 5\text{mm}$
 $H_a: \mu \neq 5\text{mm}$ (two-sided test)

$$z = \frac{5.16 - 5}{1.56 / \sqrt{100}} = 1.02$$

Normal Distribution

DECISION: The sample mean has a z-score that is NOT in the 0.05 rejection region (shown in blue). Thus, it is NOT significant at the 0.05 level.

$z = 1.02$ does NOT fall in the **Rejection Region**.

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