Topics (Book section 7-2): Sampling distribution of $\bar{X}_1 - \bar{X}_2$.

Assigned Friday, October 26.

Due Wednesday, October 31 at lecture.

*This is the last homework for Exam 2 on Thursday Nov.1

1. A random sample of size $n_1 = 16$ is selected from a normal population with a mean $\mu_1 = 80$ and a standard deviation of $\sigma_1 = 10$. A second random sample of size $n_2 = 25$ is taken from another normal population with mean $\mu_2 = 73$ and standard deviation of $\sigma_2 = 6$. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means.
   
   (a) Find $P(\bar{X}_1 - \bar{X}_2 > 4)$
   
   (b) Find $P(3.5 < \bar{X}_1 - \bar{X}_2 < 8.5)$

2. The elasticity of a polymer is affected by the concentration of a reactant. When low concentration is used, the true mean elasticity is 55, and when high concentration is used, the mean elasticity is 60. The standard deviation of elasticity is 4 regardless of concentration. If a random sample of size 16 is taken from each population, find the probability that $\bar{X}_{\text{high}} - \bar{X}_{\text{low}} \geq 8.5$.
   
   NOTE: You can assume elasticity measurements follow a normal distribution.

3. [Refresher: Material from earlier homework on $\bar{X}$.

   Household size in the United States has a mean of 2.6 people and standard deviation of 1.4 people. It should be clear that this distribution of ‘household size’ is skewed right as the smallest possible value is a household of size 1 but the largest households can be very large indeed.

   Use the central limit theorem to approximate the probability that the sample mean household size $\bar{X}$ from a random sample of size $n = 100$ is more than 3?

4. [Refresher: Material from earlier homework on normal distribution, $X$, and $\bar{X}$.

   (a) What is the 90th percentile of a normal population that has a mean of 150 and a variance of 100?

   (b) A random sample of size 25 is taken from a normal population with a mean of 150 and a variance of 100. What is the 90th percentile of the sampling distribution of $\bar{X}$?

   Recall: Averages are less variable than individual observations.

One more on the back...
5. [Refresher: Material from earlier homework on $\bar{X}$]

Let $\bar{X}_n$ be an estimator for the population mean $\mu$ in a population distributed as $N(\mu, \sigma^2)$. So, $\bar{X}_5$ is a sample mean from a random sample of size $n = 5$.

State the letter of the graphic below that coincides with the appropriate distributions for $\bar{X}_5$ and $\bar{X}_{20}$.

NOTE: All distributions below are normally distributed and generated from software, but they have different means and different variances.

A)  

B)  

C)  

D)