STA 611: Introduction to Statistical Methods
Fall 2012

Review of chapters 8 and 9

We will go through theses examples in lecture, Thursday November 29th.

1. Let $X_1, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Show that $\sum_{i=1}^n X_i^2/n$ is an unbiased estimator of σ^2.

2. The following problems from the book: 8.2.9, 8.2.10, 8.4.3, 8.6.15, 9.8.3

Other good problems

Notation: $\text{No}(\mu, \sigma^2)$ is the normal distribution.

Problem 2: The $n = 9$ independent random variables $\{X_i\}_{i \le n}$ are a simple random sample from the Normal distribution $X_i \sim \mathsf{No}(\mu, \sigma^2)$, with variance σ^2 and mean μ.

a) (7) With σ^2 unknown, describe the Rejection Region $\mathcal{R}$ for a two-sided test of size $\alpha = 0.05$ of the hypothesis

$$H_0: \ \mu = 17.5 \qquad vs. \qquad H_1: \ \mu \neq 17.5$$

Reject H_0 if:

b) (7) Same question, if variance of $\{X_i\}$ is known to be $\sigma^2 = 16$:

Reject H_0 if:

c) (6) What is the *power* $\pi(\mu)$ of the test in part b) above, for $\mu = 20$?

$\pi(20) =$ ____________

Problem 3: Once again the $n = 9$ independent random variables $\{X_i\}_{i\le n}$ are a simple random sample from the Normal distribution $X_i \sim \mathsf{No}(\mu, \sigma^2)$, with variance σ^2 and mean μ, but now we have some data. The values of a few statistics from this random sample are:

$$S(\mathbf{x}) = \sum_{i\le n} X_i = 180 \qquad T(\mathbf{x}) = \min_{i\le n} X_i = 1$$
$$V(\mathbf{x}) = \sum_{i\le n} (X_i - \bar{X})^2 = 72 \qquad W(\mathbf{x}) = \text{Median}(\{X_i\}) = 24$$

a) (7) With σ^2 unknown, give the P-value for a two-sided test of the hypothesis

$$H_0: \ \mu = 17.5 \qquad vs. \qquad H_1: \ \mu \neq 17.5$$

$P =$

b) (7) Same question, if variance of $\{X_i\}$ is known to be $\sigma^2 = 16$:

$P =$

c) (6) Are these answers consistent with your answers to parts a) and b) of Problem 2? Y N Explain.

d) With σ^2, give the 95% symmetric two-sided confidence interval for μ

For the next problem note that if p is small we expect X to be large and vice versa, so for a) you should use a test that rejects H_0 if $X \geq c$ for some c. Also, the hypotheses in a) and b) are in fact

$$a) \qquad H_0 : p \geq 1/2 \quad \text{vs.} \quad H_0 : p < 1/2$$

and

$$b) \qquad H_0 : p \leq 1/2 \quad \text{vs.} \quad H_0 : p > 1/2$$

Problem 8: A series of Statistics students answer (independently, of course) the question "Was your statistics final too long?" We record the number X of consecutive "No, it was fine" answers before the first "Well, I thought it was a little too long." Of course X has the Geometric Distribution, with probability mass function

$$f(x \mid p) = \mathsf{P}[X = x \mid p] = p\,q^x, \qquad x = 0, 1, 2, \ldots$$

We wish to test the hypothesis $H_0:\ p = 1/2$ about the probability p of a "Too long" answer. If H_0 were true, then "Yes" and "No" answers would be like Heads and Tails of a fair coin. Give all answers **exactly** and in as **simple** a form as you can. For parts a) and b) we observe $X = 3$:

a) (5) Find the P-value for $H_0 : p = 1/2$ vs. $H_1 : p < 1/2$, if $X = 3$ [First decide what outcomes are more extreme for this H_1]:

$P =$ ____________

b) (5) Now for $H_0 : p = 1/2$ vs. the other one-sided alternative $H_1 : p > 1/2$, if $X = 3$:

$P =$ ____________

Problem 8 (cont):

Now let's be more optimistic and suppose we observe $X = 10$:

c) (2) For $H_0 : p = 1/2$ vs. $H_1 : p < 1/2$, if we observe $X = 10$:

$P =$ ____________

d) (8) With a uniform prior distribution with density $\xi(p) = \mathbf{1}_{\{0<p<1\}}$, what is the posterior expectation of p if we observe $X = 10$?

$\mathsf{E}_\xi(p \mid x = 10) =$ ____________