

STA 611: Introduction to Mathematical Statistics

Instructor: Sanvesh Srivastava

TueThu 4:40pm - 5:55pm, Old Chemistry Building 116

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Introduction

About me

From India; educated at Indian Institute of Technology Kanpur (India) and Purdue University (Indiana); postdoc at SAMSI and Duke.

- Course website:
<https://stat.duke.edu/~ss602/sta611.htm>
- Homework (30%), Midterm Oct 23 (30%), Final Dec 14 (40%).
- First two homework assignments are posted on the website.
- Instructor: Sanvesh Srivastava. Office hours: Tuesdays 2:45-3:45 pm in Old Chemistry Building, 223C.
- TA: Shahryar Minhas. Office hours: Wednesdays 10:00-11:00am in Old Chemistry Building, 211A.
- TA: Yu Chen. Office hours: Wednesdays 6:15-7:15pm in Old Chemistry Building, 211A.

Make sure you have enough calculus background!

You need to be familiar with the following

- Function, limit, continuity
- Differentiation e.g. product rule, quotient rule and chain rule:

$$\begin{aligned}(f(x)g(x))' &= f'(x)g(x) + f(x)g'(x) \\ \left(\frac{f(x)}{g(x)}\right)' &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \\ (f(g(x)))' &= g'(x)f'(g(x))\end{aligned}$$

- Integration, integration techniques such as integration by parts and by substitution,

$$\begin{aligned}\int f(x)g'(x)dx &= f(x)g(x) - \int f'(x)g(x)dx \\ \int_{g(a)}^{g(b)} f(x)dx &= \int_a^b f(g(t))g'(t)dt\end{aligned}$$

First assignment

- Function of two variables, double integrals, partial derivatives, infinite series, maxima and minima of functions.
 - E.g: $\int \int f(x, y) dx dy$ over an area like $0 < x < y < 1$.

First Homework: Take the math quiz

- Before Thursday, take the math quiz on the website to help you determine if you have the necessary mathematical background.
- Solutions are also on the website.
- The quiz will not be graded.

Course overview

- Basic probability theory, Random variables, Expectations, Probability distribution functions, Convergence.
- Principles of statistical inference, Likelihood based, Frequentist and Bayesian paradigms.
- Estimation, sampling distributions and hypothesis testing.

We will cover most of the book “Probability and Statistics” By DeGroot and Schervish (Fourth edition).

- I will use a combination of slides and whiteboard. Slides will be posted on the course website.

Statistics

Statistics according to dictionary.com:

The science that deals with the collection, classification, **analysis**, and **interpretation** of numerical facts or data, and that, by use of **mathematical theories of probability**, imposes order and regularity on aggregates of more or less disparate elements.

- Collection: Design of Experiments

Statistical Analysis

- Use **probability models** to describe uncertainty
- **Statistical inference**: What does the data tell us?

Statistics and Probability

Example

- **Probability model:** If we toss a fair coin 10 times, what is the probability that we will get 5 tails?
- **Statistical inference:** If we toss a coin 10 times and get 5 tails, what does that tell us about fairness of the coin?

- Probability is the foundation of statistics

Probability

- Is there such thing as true *randomness* or *stochasticity*?
 - Is the toss of a coin “random” or completely determined (by physics)?
- Probability models are very useful for modeling uncertainty, even though the phenomenon under study is not “truly random”.
- Different interpretations of probability - no agreement on which is the “right one”
- But: There is complete agreement on the mathematical rules that probability must follow

Interpretation of Probability

Frequentist interpretation of Probability

Probability of an event is the *relative frequency* in which that event happens in the process where repeated many times under similar conditions.

- Problems with this view: How many times is needed? How similar do the conditions need to be? What about events that cannot be repeated?
- This view is sometimes called “classical statistics” and was the main stream view until recently.
- Often contributed to Jerzy Neyman and Egon Pearson and had a fierce advocate in Ronald A. Fisher.

Interpretation of Probability

Bayesian interpretation of Probability

Probability of an event is a degree of belief in whether the event will happen

- This is a subjective interpretation: Two people could have different probabilities for the same event
- Contributed to a 18th century Presbyterian Minister and mathematician Thomas Bayes.
- Avoids the philosophical difficulties of the frequency interpretation (e.g. repeatability).
- Bayesian statistics has been gaining popularity over the last few decades, largely due to advances in computing.

Remember: Regardless of interpretation, the mathematics of probability are the same - no controversy there.

Experiments / observational studies

Experiments (from book)

An **experiment** is any process where the possible **outcomes** can be identified ahead of time

- Sometimes we distinguish between *experiments* and *observational studies*:
 - In experiments we assign treatments to objects
 - In observational studies we collect data but do not interfere

Examples of outcomes of experiments / observational studies:

- Measurements from a laboratory experiment
- Results from a clinical trial
- Survival time of a cancer patient
- Collection of measurements from a sampled population

Set Theory – Notation

- **Sample Space S** : the collection (set) of possible outcomes of an experiment
 - Example: Rolling a die. $S = \{1, 2, 3, 4, 5, 6\}$
- **Event E** : a subset of S (a collection of outcomes)
 - Example: We roll an odd number on the die. $E = \{1, 3, 5\}$
- **Outcome x** : If the outcome x satisfies the conditions that define the event E we write $x \in E$
- **Empty set**: \emptyset

Interpretation of set operations

- 1 **Subset** $B \subset A$: Occurrence of B implies occurrence of A
- 2 **Intersection** $A \cap B$: Both events A and B occur
- 3 **Union** $A \cup B$: At least one of A or B occur
- 4 **Complement** A^c (or \bar{A}): the event A does not occur
- 5 A and B are **disjoint** if $A \cap B = \emptyset$. The two events cannot occur at the same time
- 6 A but not B

Some properties of set operations

- If $A \subset B$ and $B \subset A$ then $A = B$
- If $A \subset B$ and $B \subset C$ then $A \subset C$
- $(A^c)^c = A$ and $\Omega^c = \emptyset$
- Commutativity:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Some properties of set operations

- Distributive Law:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- De Morgan's laws

$$(A \cup B)^c = A^c \cap B^c \quad \text{prove this one...}$$

$$(A \cap B)^c = A^c \cup B^c$$

- Union / intersection of many sets:

$$A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n = \bigcup_{i=1}^n A_i$$

$$A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n = \bigcap_{i=1}^n A_i$$

Example (1.10 from the book)

Three six-sided dice are rolled. We define the following events:

- A = The first die shows an even number
- B = The second die shows an even number
- C = The third die shows an even number
- A_i, B_i and C_i : The first/second/third die shows the number i

Express each of the following events in terms of the named events above:

- 1 All three dice show even numbers
- 2 No die shows an even number
- 3 At least one die shows an odd number
- 4 At most two die show odd numbers
- 5 The sum of the three dice is no greater than 5