Textbook Exercises

Chapter 4

4.9  (a) 0 matches with “This event is impossible. It can never occur.”
(b) 1 matches with “This event is certain. It will occur on every trial.”
(c) 0.01 matches with “This event is very unlikely, but it will occur once in a while in a long sequence of trials.”
(d) 0.6 matches with “This event occurs more often than not.”

4.16  (a) 
\[ S = \{ \text{open}, \text{closed} \} \]

(b) 
\[ S = (0, \infty) \]

○ Notice that this is a mathematical answer. The English answer would be “the set of all positive numbers.”

○ It doesn’t matter how time is measured (seconds, minutes, hours, days, . . . .) The mathematical description for \( S \) is the same!

○ Notice also that the answer excludes the number 0. If we thought that rust could occur immediately (with no elapsed time) then the mathematical answer would instead be
\[ S = [0, \infty) \]

(c) 
\[ S = \{A, B, C, D, F\} \]

(d) It’s convenient to list the possible outcomes in \( S \):

<table>
<thead>
<tr>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAAA</td>
</tr>
<tr>
<td>AAAU</td>
</tr>
<tr>
<td>AAUA</td>
</tr>
<tr>
<td>AAUU</td>
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<td>UUAA</td>
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<tr>
<td>UUA</td>
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<tr>
<td>UUU</td>
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</tbody>
</table>
4.21  • Model 1 - illegitimate since probabilities don’t add to 1.0
• Model 2 - legitimate
• Model 3 - illegitimate since probabilities don’t add to 1.0
• Model 4 - illegitimate since probabilities don’t add to 1.0 and some of them exceed 1.0

4.22  (a) The probabilities add to one, as required.
(b) 0.09
(c) 0.70

4.23  (a) 0.48
(b) 0.09

Chapter 5

5.17  \[ P(A \text{ or } B) = 0.6 + 0.5 - 0.3 = 0.8 \]

5.18

Events:
A = like country music
B = like gospel music

Available Probabilities:
P(A) = 0.40
P(B) = 0.30
P(A and B) = 0.10

Answers:

(a) Method 1: Use Addition Rule:

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
= 0.40 + 0.30 - 0.10
= 0.60 = 60\%
\]

or Method 2: Use the table shown below:

\[
P(A \text{ or } B) = P(A \text{ and } B) + P(A \text{ and } \bar{B}) + P(\bar{A} \text{ and } B)
= 10\% + 30\% + 20\%
= 60\%
\]

(b) \[P(A \text{ and } \bar{B}) = ?\]

\[
\begin{array}{c|c|c}
A & \bar{A} & \\
\hline
\bar{B} & 10\% & 30\% \\
B & 40\% & 100\%
\end{array}
\implies
\begin{array}{c|c|c|c}
A & \bar{A} & \\
\hline
\bar{B} & 10\% & 20\% & 30\% \\
B & 30\% & 40\% & 70\%
\end{array}
\]

so \[P(A \text{ and } \bar{B}) = 0.30 = 30\%\]
5.16

Events:
H = admitted to Harvard
S = admitted to Stanford

Available Probabilities:
P(H) = 0.4
P(S) = 0.5
P(H and S) = 0.2

Answers:

(b) P(\overline{H} and \overline{S}) = ?

<table>
<thead>
<tr>
<th></th>
<th>(H)</th>
<th>(\overline{H})</th>
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</thead>
<tbody>
<tr>
<td>S</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>\overline{S}</td>
<td>0.4</td>
<td>1.0</td>
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</tbody>
</table>

\(\Rightarrow\)

<table>
<thead>
<tr>
<th></th>
<th>(H)</th>
<th>(\overline{H})</th>
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</thead>
<tbody>
<tr>
<td>S</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>\overline{S}</td>
<td>0.4</td>
<td>0.6</td>
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</tbody>
</table>

so \(P(\overline{H} and \overline{S}) = 0.30\)

(c) \(P(\overline{H} and S) = 0.30\)

5.6

Events:
A = prosperous
B = educated

Available Probabilities:
P(A) = 0.180
P(B) = 0.290
P(A and B) = 0.105

Answers:

(a) 0.365
(b) 0.185
(c) 0.635
(d)

\[
P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.105}{0.180} = 0.583
\]

(e)

\[
P(B|\overline{A}) = \frac{P(\overline{A} \text{ and } B)}{P(\overline{A})} = \frac{0.185}{0.820} = 0.226
\]

(f)

\[
P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.105}{0.290} = 0.362
\]
(g) Does prosperous depend on educated?

Does A depend on B?

Yes since \( P(A|B) = 0.362 \neq 0.180 = P(A) \).

(h) It increases the chances from 18.0% to 36.2%.

5.25

\[
P(A \text{ and } B) = 0.1472
\]

5.26

(a) 0.32
(b) 0.08

5.27

(a) 0.6946
(b) 0.7627
(c) 0.2921
(d) 0.0158
(e) 0.1884

(continued next page)
Additional Problems

1. (a) There are 36 outcomes:

<table>
<thead>
<tr>
<th>D₁</th>
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<tr>
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</table>

(b) \( P(\text{total is 7}) = \frac{6}{36} = \frac{1}{6} \)
(c) \( P(\text{total is 4}) = \frac{3}{36} = \frac{1}{12} \)
(d) \( P(\text{at least one die is 4}) = \frac{11}{36} \)
(e) \( P(\text{both dice less than 3}) = \frac{4}{36} = \frac{1}{9} \)

2. (a) Persons: \( W₁, W₂, W₃, B₁, H₁, H₁ \)

There are 15 outcomes:

<table>
<thead>
<tr>
<th>#1</th>
<th>#2</th>
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<tbody>
<tr>
<td>( W₁ )</td>
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<tr>
<td>( W₁ )</td>
<td>( W₃ )</td>
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<td>( H₁ )</td>
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</tbody>
</table>

(b) Yes, each outcome has probability \( \frac{1}{15} \).
(c) \( P(\text{at least one hispanic}) = \frac{9}{15} = \frac{3}{5} = 0.60 \)
(d) \( P(\text{black}) = \frac{5}{15} = \frac{1}{3} = 0.333 \)
(e) \( P(\text{blacks or hispanics}) = \frac{12}{15} = \frac{4}{5} = 0.80 \)
(f) \( P(\text{only whites}) = 1 - P(\text{blacks or hispanics}) = 1 - .8 = 0.20 \)