Textbook Exercises

Chapter 5

5.12 \( P(\text{hire VP}) = 0.6, P(\text{hire all three managers}) = (0.8)^3 = 0.512. \) Offer job to the VP candidate.

5.13 \( P(\text{win at least once}) = 1 - P(\text{lose all five times}) = 1 - (0.98)^5 = 0.0961 \)

5.28 (a) 0.5764
   (b) 0.50
   (c) No, since \( P(\text{woman}) = 0.5764 \neq 0.50 = P(\text{woman} | \text{professional}) \)

5.49 (a)
   - \textbf{Events}
     - \( A = \text{credit card holder defaults on payments} \)
     - \( B = \text{credit card holder is late with two or more monthly payments} \)
   - \textbf{Available Probabilities}
     - \( P(B | A) = 0.88 \)
     - \( P(A) = 0.03 \)
     - \( P(B | A^c) = 0.40 \)
   - \textbf{Answer}
     - 0.0637
   (b) About 94
   (c) No. The vast majority (93.63%) of customers who are late with payments do not default.

5.50 0.8

5.51 Yes! There are two reasons which can be given to prove the answer. (Giving either is fine):
   - \( P(B | A) = 0.5 = P(B) \)
   - \( P(A | B) = 0.6 = P(A) \)

5.52
   (a) \( P( A \text{ and } B ) = 0.3 \)
   (b) \( P( A \text{ and } B^c ) = 0.3 \)
   (c) \( P( A^c \text{ and } B ) = 0.2 \)
   (d) \( P( A^c \text{ and } B^c ) = 0.2 \)

(continued)
This is a challenging problem. To get started, use the Three Steps from Notebook page 73:

**Events:**
A = nonconforming product
B = completely-inspected product

**Available Probabilities:**
P(A) = 0.08
P(B|A) = 0.55
P(B|A) = 0.20

**Answers:**

(a)

\[ P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \]

Then

\[ P(A \text{ and } B) = P(A)P(B|A) \]
\[ = (0.08)(0.55) \]
\[ = 0.044 \]

How to find P(B)? Break “B” into two parts!

\[ P(B) = P(A \text{ and } B) + P(\bar{A} \text{ and } B) \]

and

\[ P(\bar{A} \text{ and } B) = P(\bar{A})P(B|\bar{A}) \]
\[ = (0.92)(0.20) \]
\[ = 0.184 \]

So

\[ P(B) = P(A \text{ and } B) + P(\bar{A} \text{ and } B) \]
\[ = 0.044 + 0.184 \]
\[ = 0.228 \]

and

\[ P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \]
\[ = \frac{0.044}{0.228} \]
\[ = 0.1930 \]

(b) From part (a),

\[ P(B) = 0.228 \approx 22.8\% \]
1. (a) \[ P(\text{accident}) = P(\text{accident and side airbags}) + P(\text{accident and no side airbags}) \]
\[ = 0.03 + 0.12 = 0.15 \]
(b) \[ P(\text{no accident and side airbags}) = 0.40 \]
(c) Define events:
\[ A: \text{accident} \]
\[ B: \text{side airbags} \]
Then \( P(A) = 0.15 \)
\[ P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.03}{0.43} = 0.0698 \]
Yes, Event A depends on Event B since \( P(A|B) = 0.0698 \neq 0.15 = P(A) \).
(d) Yes because the use of side airbags reduces the probability of an accident from \( P(A) = 15\% \) to \( P(A|B) = 6.98\% \).

2. (a) The probability that the first car has side airbags is 0.43, as given in the problem. If the first car has side airbags, the probability that the second car has side airbags is still approximately 0.43 since the number of cars from which we are choosing is so large.

(b) Define the events
\[ A = \text{first car has side airbags} \]
\[ B = \text{second car has side airbags} \]
The English answer to part (a) can be expressed mathematically as
\[ P(B|A) = 0.43 \]
If we know nothing about the first car, the chances for the second car are the same:
\[ P(B) = 0.43 \]
Therefore \( P(B|A) = 0.43 = P(B) \) so events A and B are independent.

(c) Define a third event C: Third car has side airbags.
Continuing the logic from (b), all three events A, B, C are independent.
\[ P(\text{A and B and C}) = P(\text{ABC}) = P(A) \times P(B) \times P(C) = (0.43)^3 = 0.0795 \]
(d) Exactly one car has side airbags implies one of three possible patterns:
\[ \overline{ABC} \text{ or } \overline{AB}C \text{ or } \overline{A}BC \]
By independence,
\[ P(\overline{A}BC) = P(A) \times P(\overline{B}) \times P(C) \]
\[ = (0.43)(0.57)(0.57) \]
\[ = (0.43)(0.57)^2 = 0.1397 \]
So \( P(\text{one car}) = P(\overline{A}BC) + P(\overline{AB}C) + P(ABC) = 3 \times (0.1397) = 0.4191 \)
3. (a) \( P(\text{Good}) = 0.3800 + 0.2375 + 0.3325 = 0.95 \)

\[
P(\text{Good} \mid \text{Machine 1}) = \frac{P(\text{Good and Machine 1})}{P(\text{Machine 1})} = \frac{0.3800}{0.4000} = 0.95
\]

Therefore, whether an item is good does not depend on whether the item is produced by Machine 1. Similar calculations and conclusions hold for Machines 2 and 3.

(b) The quality of items produced is not related to choice of machines. Stated differently, the three machines are equally effective in producing good items.

4.

(a) 2.71%

(b) 11.69% So the chance for Tucson more than quadruples compared to Coralville, even through the staff is only twice as large. (That’s not fair to the good folks in Coralville!)

(c) 41.14% So again the chance almost quadruples when the number of staff members merely doubles (from Tucson to Honolulu.)

(d) \( n = 23 \)

5.

(a) \( P(\text{more than 100 hours} \mid \text{more than 60 hours}) = 0.6190 \)

(b) Choose the used light bulb since the probability of success is greater:

\[
P(\text{more than 100 hours} \mid \text{more than 60 hours}) = 0.6190 > 0.52 = P(\text{more than 100 hours})
\]

6.

(a) \( P(\text{red}) = \frac{16}{30} = 0.5333 \)

(b) \( P(\text{Box A} \mid \text{red}) = \frac{3}{16} = 0.1875 \)

(c) \( P(\text{Box B} \mid \text{red}) = \frac{5}{16} = 0.3125 \)

(d) \( P(\text{Box C} \mid \text{red}) = \frac{8}{16} = 0.5 \)

(e) 100%

7. \( P(\text{at least two match}) = 0.625 \)

8. (a) 0.56

(b) 0.3762

(c) 0.088