6.2 Introduction to Probability

part 2

- From Tables to Probability
  - Computing relative frequency probabilities from a table.
- Probability Distributions

Summary Table

- The table below provides information on the hair color and eye color of 592 statistics students (The American Statistician, 1974).

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Row Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brown</td>
</tr>
<tr>
<td>Black</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>106</td>
</tr>
<tr>
<td>Brown</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>286</td>
</tr>
<tr>
<td>Red</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>71</td>
</tr>
<tr>
<td>Blond</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>127</td>
</tr>
<tr>
<td></td>
<td>592</td>
</tr>
</tbody>
</table>

- If a student is drawn at random from this population...
  - What is the probability that they are blond?
    - ANS: 127/592 = 0.2145
  - What is the probability that they are NOT blond?
    - ANS: P(NOT blond) = 1-P(blond) =1-127/592 = 0.7855
    - ANS: (71 +286 + 108)/592 = 465/592 = 0.7855
If a student is drawn at random from this population...
- What is the probability that they are blond and have brown eyes?
  - Ans: $\frac{7}{592} = 0.0118$
- What is the probability that they have red hair and have green eyes?
  - Ans: $\frac{14}{592} = 0.0236$

Based on this table...
- What is the probability that a blond student has brown eyes?
  - Ans: $\frac{7}{127} = 0.0551$
- What is the probability that a brown-haired student has hazel eyes?
  - Ans: $\frac{54}{286} = 0.1888$

**Relative Frequency Method**

- **Step 1.** Repeat or observe a process many times and count the number of times the event of interest, $A$, occurs.
- **Step 2.** Estimate $P(A)$ by
  
  $P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{total number of observations}}$
Example: 500-year flood

Geological records indicate that a river has crested above a particular high flood level four times in the past 2,000 years. What is the relative frequency probability that the river will crest above the high flood level next year?

\[
\text{number of years with flood} \quad \frac{4}{2000} = \frac{1}{500}
\]

The probability of having a flood of this magnitude in any given year is 1/500, or 0.002.

A probability distribution shows the possible values that could occur, and the probability for each occurring.

The possible values can be seen on the horizontal axis.
A probability distribution shows the possible values that could occur, and the probability for each occurring.

The probability of each value is shown by the height of the bar.

Example: Roll two 6-sided dice

Find the probability distribution for the 'sum of two dice'.

Most likely sum is a sum of 7.

Only one way to get sum of 12 (double 6's).

Recall, \( P(\text{sum of 11}) = \frac{2}{36} = 0.0556 \)

And that is the height of the bar above the number "11".
Example: Roll one 6-sided dice

- Find the probability distribution for ‘the number that turns up’.

Each of 6 values is equally likely, at \( \frac{1}{6} = 0.1667 \)

This is a uniform distribution.

Probability Distributions

- How to make a probability distribution.
  1) List all possible outcomes. Use a table or figure it is helpful.
  2) Identify outcomes that represent the same event. Find the probability of each event.
  3) Make a table in which one column lists each event and another column lists each probability. The sum of all the probabilities must be 1.

Example: Making a probability distribution

- Random experiment: Toss Three Coins
- Make a probability distribution for the number of heads that occurs.
  1) List the possible outcomes (equally likely):
     HHH, HHT, HTH, THH, HTT, THT, TTH, TTT
  2) Identify outcomes that represent the same event. There are four possible events: 0, 1, 2, or 3 heads.
Example: Making a probability distribution

- Random experiment: Toss Three Coins
- Make a probability distribution for the number of heads that occurs.

3) Make a table

<table>
<thead>
<tr>
<th>Number of heads</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/8</td>
</tr>
<tr>
<td>1</td>
<td>3/8</td>
</tr>
<tr>
<td>2</td>
<td>3/8</td>
</tr>
<tr>
<td>3</td>
<td>1/8</td>
</tr>
</tbody>
</table>

This column sums to 1.