STAT:2020
Homework 3 (week 3)
Topics: Bayes Theorem, Discrete Random Variables (probability distributions, cumulative distributions, mean and variance), discrete uniform distribution and binomial distribution.

Assigned Wednesday, September 7.
Due Thursday, September 15 at 4:25pm for all\textsuperscript{1} students.

- Students can turn-in HW at lecture (on Wednesday).
- Otherwise, students will need to turn it in at 241 SH (statistics office) by 4:25pm. Ask for Margie. You can also turn it in at my office 211 SH (under the door is fine).

\textsuperscript{1} Sections B22 and B24 can turn in HW at discussion.

Do all parts of following book problems unless otherwise stated.
3-4
3-18
3-24 Notice that each distinct part is either correctly classified or misclassified, and that there is more than one way to come up with a total of 1 misclassification (such as MCC, CMC, CCM).
3-40
3-49
3-60
3-81
3-94
3-106

Do the following written problems:
3A-1 The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.98. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person actually has the disease?

More on back...
3A-2 The following table presents the probability mass function of the number of defects \( X \) in a randomly chosen printed-circuit board.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0.55</td>
<td>0.25</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a) Find \( E(X) \).

b) Find \( E(X^2) \).

c) Find \( V(X) \) which is the variance of \( X \).

3A-3 The probability mass function for the number of text messages, \( X \), you receive in a day from your friend Charlie is shown below (he’s the only friend who is currently texting you).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0.10</td>
<td>0.20</td>
<td>0.40</td>
<td>0.20</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The total cost of the received texts on that day depends on the number of texts you receive (and the company charges you less per text when you have a lot of them). Let \( h(X) \) represent the total daily cost of received texts in a day where

\[
h(X) = \begin{cases} 
0.15X & \text{if } X \leq 2 \\
0.10 + 0.10X & \text{if } X > 2 
\end{cases}
\]

Find \( E[h(X)] \) \{the expected value of the total daily cost of Charlie texting you.\}.

3A-4 A system consists of four components connected as shown in the following diagram.

![Diagram of a system with four components](image)

Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.08, 0.06, 0.08, and 0.20, respectively, what is the probability that the system functions? \( \text{(Hint: work this problem in three distinct steps, each of which was discussed in class.)} \)