**Example:** A balanced two-way ANOVA without significant interaction

* Fishing Data

- Objective: to see if sinker weight and/or rod length affect the distance the line is cast, and if the rod effect (if present) is the same for differing sinker weight (i.e. see if there is interaction).

- Variables:
  - Distance: distance line cast in meters
  - Sinker: factor with levels: 8 oz./12 oz.
  - Rod: factor with levels: 6 ft./7 ft.

---

```r
# Re-assign the variables to be 'factors':
> rod = as.factor(rod)
> sinker = as.factor(sinker)

> is.factor(sinker)
[1] TRUE

> is.factor(rod)
[1] TRUE

> sinker
[1] 1 1 1 1 1 1 2 2 2 2 2 1 1 2 2
Levels: 1 2

> rod
[1] 1 1 1 2 2 2 2 2 2 2 1 1 2 1 1
Levels: 1 2

> nrow(rod sinker)
[1] 16

> table(sinker, rod)
  rod  
sinker 1 2
     1 4 4
     2 4 4

This is a balanced two-way ANOVA.
```

---

**Example:** Fishing data

```r
> rodsink = read.csv("rod_and_sinker.csv")
> attach(rod sinker)
> head(rod sinker)
  rod sinker distance order
1 1 1 1 28.0 1
2 1 1 30.5 2
3 1 1 31.0 6
4 1 1 30.0 5
5 2 1 33.5 9
6 2 1 35.0 10

> is.factor(rod)
[1] FALSE

> is.factor(sinker)
[1] FALSE
```

The variables `rod` and `sinker` were entered as numbers in the data file rather than letters, so we have to tell R that these are factors (or it will see them as continuous data).

If rod=1 ⇒ 6 foot rod
If sinker=1 ⇒ 8 oz. sinker

---

Get the interaction plot of the means:

```r
> interaction.plot(sinker, rod, distance)
```

---

First factor listed is represented in the `rows`.
Get a more sophisticated plot:

```r
> plot(c(0.75,2.25),range(distance),type="n",
     xlab="Sinkers",ylab="Effectiveness",axes=F)
> axis(2)
> axis(1,at=1:2,c("8 ounces","12 ounces"))
> box()
> points(jitter(as.numeric(sinker[rod==1]),factor=.15),
         distance[rod==1],pch="1",col=1)
> points(jitter(as.numeric(sinker[rod==2]),factor=.15),
         distance[rod==2],pch="2",col=2)
> lines(c(1,2),means[,1],col=1)
> lines(c(1,2),means[,2],col=2)
> legend(1,42,c("Rod: 6 feet","Rod: 7 feet"),
       col=c(1,2),lty=1)
```

As the primitive interaction plot also suggested, there may be some slight interaction (we’ll test for this). But even if there is interaction, having a rod of 7 feet is better at both levels of sinker weights.

Before fitting the linear model, we will change our dummy regressor coding to the *sum-to-zero* constraint.

```r
> contrasts(rod)
  [,1]
2 1 0
1 0 1
2 1 0

## Reassign as sum-to-zero contrast dummy regressors:
> contrasts(rod)=contr.sum(levels(rod))

> contrasts(sinker)
  [,1]
2 1 0
1 0 1

## Reassign as sum-to-zero contrast dummy regressors:
> contrasts(sinker)=contr.sum(levels(sinker))
```

Fit the linear model with interaction:

```r
> lm.out=lm(distance ~ sinker + rod + sinker:rod)
> summary(lm.out)
```

Coefficients:                         Estimate  Std. Error    t value  Pr(>|t|)
(Intercept)           35.18750      0.51354   68.526     <2e-16 ***
sinker1              -2.37500      0.51354    -4.625     0.000585 ***
rod1                 -2.43750      0.51354    -4.747     0.000475 ***
sinker1:rod1          -0.50000      0.51354    -0.974     0.349411

Residual standard error: 2.054 on 12 degrees of freedom
Multiple R-squared: 0.7890, Adjusted R-squared: 0.7363
F-statistic: 14.96 on 3 and 12 DF,  p-value: 0.0002341

Items provided by the ‘summary’ output:

1) Overall F-test (reject, thus something in the model is useful for explaining the variability in distance).

2) *Test for dummy regressors. In this case, since there is only 1 dummy regressor for each factor, this gives us the tests for main effects of *sinker* and *rod*.  

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We can also use the Anova(lm.out,type= "III") command to get the appropriate tests for main effects and interaction:

```r
> Anova(lm.out,type="III")
Anova Table (Type III tests)

Response: distance

<table>
<thead>
<tr>
<th>Sum Sq</th>
<th>Df</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>19810.6</td>
<td>1</td>
<td>4695.837</td>
</tr>
<tr>
<td>sinker</td>
<td>90.3</td>
<td>1</td>
<td>21.3926</td>
</tr>
<tr>
<td>rod</td>
<td>95.1</td>
<td>1</td>
<td>22.5333</td>
</tr>
<tr>
<td>sinker:rod</td>
<td>4.0</td>
<td>1</td>
<td>0.9481</td>
</tr>
<tr>
<td>Residuals</td>
<td>50.6</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

---

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

Items provided by the `Anova(lm.out,type= "III")` output:

1) \( F \)-tests for the main effects and interaction. (You don’t get an overall \( F \)-test here.)

2) Type III sums of squares for each term.

\( \Rightarrow \) there is NOT significant interaction.

Since there was not significant interaction, we can use the simpler ‘additive’ model for any further analysis.

```r
> Anova(lm.out.red,type="III")
Anova Table (Type III tests)

Response: distance

<table>
<thead>
<tr>
<th>Sum Sq</th>
<th>Df</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>4920.8</td>
<td>1</td>
<td>1171.071</td>
</tr>
<tr>
<td>sinker</td>
<td>90.3</td>
<td>1</td>
<td>21.478</td>
</tr>
<tr>
<td>rod</td>
<td>95.1</td>
<td>1</td>
<td>22.624</td>
</tr>
<tr>
<td>Residuals</td>
<td>54.6</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

---

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

Both sinker weight and rod length have a significant effect on distance cast.

What are the effects of these factors?

In other words, is it better to have a 6 or 7 foot rod? Or an 8 or 12 oz. sinker?

As we mentioned earlier, in a two-way ANOVA that includes interaction, a common first step is to test for interaction.

We can get this from the ‘Anova’ output, or we can also do a partial \( F \)-test.

Using partial \( F \)-test for interaction term:

```r
> lm.out.red=lm(distance ~ sinker + rod)
> anova(lm.out.red,lm.out)

Analysis of Variance Table

Model 1: distance ~ sinker + rod
Model 2: distance ~ sinker + rod + sinker:rod

<table>
<thead>
<tr>
<th>Res.Df</th>
<th>RSS</th>
<th>Df</th>
<th>Sum of Sq</th>
<th>F</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>54.6</td>
<td>13</td>
<td>0.0004675</td>
<td>0.9481</td>
<td>0.3494</td>
</tr>
</tbody>
</table>

This is the same \( p \)-value as for the dummy regressor called sinker2:rod2 in the summary output, just square the \( t \), \((−0.974)^2 \approx 0.948\), to get the \( F \). And the same \( p \)-value for interaction using the ‘Anova’ command.

We can plot the fitted model by getting the 4 predicted cell means.

Recall how is \textbf{R} coding the dummy regressors:

```r
contrasts(rod) # [,1] 1 1 2 -1
contrasts(sinker) # [,1] 1 1 2 -1

lm.out.red$coefficients

(Intercept) sinker1 rod1
35.1875 -2.3750 -2.4375
```

The coefficients are given for sinker=1 and rod=1.

```r
rod

<table>
<thead>
<tr>
<th>sinker7</th>
<th>35.1875 - 2.3750 + 2.4375</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>35.1875 - 2.3750 + 2.4375</td>
</tr>
</tbody>
</table>

<p>|</p>
<table>
<thead>
<tr>
<th>sinker7</th>
<th>35.1875 + 2.3750 + 2.4375</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>35.1875 + 2.3750 + 2.4375</td>
</tr>
</tbody>
</table>

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```r
rod

<table>
<thead>
<tr>
<th>sinker7</th>
<th>35.1875 - 2.3750 + 2.4375</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>35.1875 - 2.3750 + 2.4375</td>
</tr>
</tbody>
</table>

<p>|</p>
<table>
<thead>
<tr>
<th>sinker7</th>
<th>35.1875 + 2.3750 + 2.4375</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>35.1875 + 2.3750 + 2.4375</td>
</tr>
</tbody>
</table>
```

We can plot the fitted model by getting the 4 predicted cell means.
The four fitted values:
> unique(lm.out.red$fitted)
[1] 30.375 35.250 40.000 35.125

Aligned with the data:
> cbind(rod,sinker,lm.out.red$fitted)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>28.0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>30.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>31.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>30.5</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>33.5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>35.0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>38.0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>37.5</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2</td>
<td>42.0</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2</td>
<td>40.5</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>2</td>
<td>33.0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>34.0</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>1</td>
<td>36.0</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>2</td>
<td>38.5</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>2</td>
<td>38.0</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>2</td>
<td>37.5</td>
</tr>
</tbody>
</table>

CONCLUSIONS:
- The heavier sinker has a positive effect on distance.
- The longer rod has a positive effect on distance.

The fitted model shows parallel profile plots.
> plot(c(0.75,2.25),range(distance),type="n",xlab="Sinker",ylab="distance",axes=F,cex.lab=2,cex.axis=2)
> axis(2)
> axis(1,at=1:2,c("8 ounces","12 ounces"))
> box()
> points(jitter(as.numeric(sinker[rod==1]),factor=.15),distance[rod==1],pch="1",col=1,cex=1.5)
> points(jitter(as.numeric(sinker[rod==2]),factor=.15),distance[rod==2],pch="2",col=2,cex=1.5)

## lines connecting the relevant fitted values:
> lines(c(1,2),c(30.375,35.125),col=1)
> lines(c(1,2),c(35.250,40.000),col=2)
> legend(1,42,c("Rod: 6 feet","Rod: 7 feet"),col=c(1,2),lty=1,cex=1.5)

- The positive effects are seen in the plot, and also in the estimated coefficients in the effects model.
  \[ Y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk} \]
  \[ \hat{\alpha}_2 = 2.3250 \quad (\text{and} \; \hat{\alpha}_1 = -2.3250) \]
  \[ \hat{\beta}_2 = 2.4375 \quad (\text{and} \; \hat{\beta}_1 = -2.4375) \]
  \[ \hat{\mu} = 35.1875 \]

> mean(distance)  ## Y-bar = mu.hat because balanced.
[1] 35.1875

- As there was no interaction, the rod effect is the same for both levels of sinker.
- The fitted model shows parallel lines.

RECOMMENDATION FROM THIS DATA: For the longest distance, get a rod of length 7 ft. and sinker that weighs 12 oz.