How are the t distribution and F distribution related?

t - distributions:

F - distributions:
If you square a $t_{df}$ you get an $F(1, df)$.

i.e. you get an $F$ with 1 degree of freedom in the numerator and $df$ degrees of freedom in the denominator.

Let's consider a $t$ from a two-sample t-test.

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Under $H_0 : \mu_1 = \mu_2$,

$t$ is distributed as $t_{n_1+n_2-2}$
Let’s square the $t$:

$$t^2 = \left( \frac{\bar{Y}_1. - \bar{Y}_2.}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right)^2$$

$$= \frac{(\bar{Y}_1. - \bar{Y}_2.)^2}{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = \frac{n_1 n_2 (\bar{Y}_1. - \bar{Y}_2.)^2}{S_p^2 (n_1 + n_2)}$$

$$= \frac{n_1 n_2 (n_1 + n_2) (\bar{Y}_1. - \bar{Y}_2.)^2}{S_p^2 (n_1 + n_2)^2}$$

$$= \frac{n_1 n_2^2 (\bar{Y}_1. - \bar{Y}_2.)^2 + n_1^2 n_2 (\bar{Y}_1. - \bar{Y}_2.)^2}{S_p^2 (n_1 + n_2)^2}$$

$$= \frac{n_1 (\bar{Y}_1. - \bar{Y}.)^2 + n_2 (\bar{Y}_2. - \bar{Y}.)^2}{S_p^2}$$
\[
\frac{\sum_{i=1}^{2} n_i \left( \bar{Y}_i - \bar{Y}_. \right)^2}{\frac{RSS}{n-2}} = \left( \frac{\sum_{i=1}^{2} n_i (\bar{Y}_i - \bar{Y}_.)^2}{1} \right) \frac{RSS}{n-2}
\]

\[
= \left( \frac{SS_{\text{group}}}{1} \right) \frac{RSS}{n-2} = \frac{MS_{\text{group}}}{MSE} = F
\]

where \( F \) is distributed as \( F(1, n_1 + n_2 - 2) \) and \( F \) is the overall F-test in one-way ANOVA with 2 groups.