Simple example for test of nonadditivity

This is the 3-factor experiment in unassigned Problem 8.2.

```sas
SAS> data pboard;
SAS> infile "pr08-2.dat";
SAS> input slash target resin response;

First try the model without the 3-factor interaction:

SAS> proc glm data=pboard;
SAS> class slash target resin;
SAS> model response = slash target resin
SAS> slash*target slash*resin target*resin / ss1;
SAS> output out = diagnost r=resids p=fitted;
SAS> proc gplot data=diagnost;
SAS> plot resids*fitted;

The GLM Procedure
Dependent Variable: response

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>13</td>
<td>171.6122222</td>
<td>13.2009402</td>
<td>28.61</td>
<td>0.0027</td>
</tr>
<tr>
<td>Error</td>
<td>4</td>
<td>1.8455556</td>
<td>0.4613889</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>17</td>
<td>173.4577778</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>slash</td>
<td>2</td>
<td>9.7377778</td>
<td>4.8688889</td>
<td>10.55</td>
<td>0.0254</td>
</tr>
<tr>
<td>target</td>
<td>1</td>
<td>105.6088889</td>
<td>105.6088889</td>
<td>228.89</td>
<td>0.0001</td>
</tr>
<tr>
<td>resin</td>
<td>2</td>
<td>52.1244444</td>
<td>26.0622222</td>
<td>56.49</td>
<td>0.0012</td>
</tr>
<tr>
<td>slash*target</td>
<td>2</td>
<td>0.8844444</td>
<td>0.4422222</td>
<td>0.96</td>
<td>0.4570</td>
</tr>
<tr>
<td>slash*resin</td>
<td>4</td>
<td>2.6655556</td>
<td>0.6663889</td>
<td>1.44</td>
<td>0.3652</td>
</tr>
<tr>
<td>target*resin</td>
<td>2</td>
<td>0.5911111</td>
<td>0.2955556</td>
<td>0.64</td>
<td>0.5737</td>
</tr>
</tbody>
</table>

The residual plot looks OK, but all the 2-way interactions are nonsignificant, so let’s pool them into error…

SAS> proc glm data=pboard;
SAS> class slash target resin;
```
SAS> model response = slash target resin / ss1;
SAS> output out = diagnost r=resids p=fitted;
SAS> proc gplot data=diagnost;
SAS> plot resids*fitted;

The GLM Procedure  
Dependent Variable: response  

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<tr>
<th>Source</th>
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<th>Sum of Squares</th>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>167.4711111</td>
<td>33.4942222</td>
<td>67.14</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>5.9866667</td>
<td>0.4988889</td>
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<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>17</td>
<td>173.4577778</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<tr>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>slash</td>
<td>2</td>
<td>9.7377778</td>
<td>4.8688889</td>
<td>9.76</td>
<td>0.0030</td>
</tr>
<tr>
<td>target</td>
<td>1</td>
<td>105.6088889</td>
<td>105.6088889</td>
<td>211.69</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>resin</td>
<td>2</td>
<td>52.1244444</td>
<td>26.0622222</td>
<td>52.24</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

To get the nonadditivity test, add a new variable “nonadd” equal to the square of the fitted values. Then incorporate that into the model.

SAS> data pboard;
SAS> set diagnost; /* use the variables in diagnost */
SAS> nonadd = fitted**2;
SAS> proc glm data=pboard;
SAS> class slash target resin;
SAS> model response = slash target resin nonadd / ss1 solution;

The GLM Procedure  
Dependent Variable: response  

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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
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<td>167.4905844</td>
<td>27.9150974</td>
<td>51.46</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>11</td>
<td>5.9671934</td>
<td>0.5424721</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>17</td>
<td>173.4577778</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Source | DF | Type I SS | Mean Square | F Value | Pr > F
---|---|---|---|---|---
slash | 2 | 9.7377778 | 4.8688889 | 8.98 | 0.0049
target | 1 | 105.6088889 | | 194.68 | <.0001
resin | 2 | 52.1244444 | 26.0622222 | 48.04 | <.0001
nonadd | 1 | 0.0194733 | 0.0194733 | 0.04 | 0.8532

Standard Parameter Estimate Error t Value Pr > |t|
Intercept | 60.66105333 B | 48.88166926 | 1.24 | 0.2404
slash 1 | -2.38633591 B | 3.12375836 | -0.76 | 0.4610
slash 2 | -1.10762706 B | 1.50888108 | -0.73 | 0.4783
slash 3 | 0.00000000 B | | | |
target 1 | -6.42324388 B | 8.34013025 | -0.77 | 0.4574
target 2 | 0.00000000 B | | | |
resin 1 | -5.47584579 B | 7.09852602 | -0.77 | 0.4567
resin 2 | -2.13398197 B | 2.85025487 | -0.75 | 0.4697
resin 3 | 0.00000000 B | | | |
nonadd | -0.00352789 | 0.01862021 | -0.19 | 0.8532

Note that nonadd is nonsignificant in this model; so we deem the additive model OK, and proceed with some follow-up tests—see below. (Important: Do not include nonadd in the final model! It’s for diagnostic purposes only—not modeling.)

Had the nonadditivity been significant, the suggested transformation would be to the power of 1 – its coefficient, or the log transformation if the coefficient is near 1.

Our follow-up analysis considers the marginal effects for each factor, since there are no significant interactions. Both slash and resin are quantitative variables, so I consider linear and quadratic contrasts. For target, I’ll estimate the two means and the difference between them. I’m electing to use estimate statements for the contrasts for which I can provide meaningful interpretations, and contrast statements for things I just want to test.

```
SAS> proc glm data=pboard;
SAS> class slash target resin;
SAS> model response = slash target resin;
SAS> estimate "slashLin" slash -1 0 1 / divisor = 50;
SAS> contrast "slash**2" slash 1 -2 1;
SAS> estimate "resinLin" resin -1 0 1 / divisor = 6;
SAS> contrast "resin**2" resin 1 -2 1;
SAS> estimate "target42" intercept 3 target 3 0 slash 1 1 1 resin 1 1 1 / divisor=3;
SAS> estimate "target48" intercept 3 target 0 3 slash 1 1 1 resin 1 1 1 / divisor=3;
SAS> estimate "targetdiff" target -1 1;
```

The divisors for the linear effects are based on the fact that the slash levels are 0, 25, and 50 (a range of 50); and the resin levels are 6, 9, 12 (a range of 6). That way, I’m estimating slopes. Note in the estimate statements for the target means, I gave equal weights of $\frac{1}{3}$ to each of the three levels of slash and the three levels of resin. The results are on the next page.
Neither of the quadratic effects are significant. So I’ll be happy describing them by their linear effects. The main conclusions are:

- For each 1% increase in slash density, the board density increases by .036 lb/ft$^3$ on average, plus or minus about .017 (with 95% confidence—I multiplied the SE by $t_{.025,12} = 2.18$).
- For each 1% increase in resin, the board density increases by about .69 lb/ft$^3$ on average, plus or minus about .14.
- When we were trying to get a density of 42 or 48, respectively, we actually achieved mean densities of 43.77 ± .47 and 48.61 ± .47; the difference is 4.84 ± .66.

Additional note  In case you are uncomfortable with, or don’t believe that I estimated the slopes correctly, an ordinary regression model (i.e., not specifying slash or resin as class variables) will yield the same estimates. The standard errors are slightly different because this model doesn’t include the quadratic effects.