One-factor (dosage) experiment with five levels of time equally spaced (balanced design). Response: Concentration of solution.

This is a completely randomized design (CRD).

(*Applied Linear Statistical Models, 5th ed. by Kutner et al.*)

Data input for solution concentration data set

```plaintext
data orthopolyex; input concentration time;
cards;
0.07 9.0
0.09 9.0
0.08 9.0
0.16 7.0
0.17 7.0
0.21 7.0
0.49 5.0
0.58 5.0
0.53 5.0
1.22 3.0
1.15 3.0
1.07 3.0
2.84 1.0
2.57 1.0
3.10 1.0
;

data orthopolyex; set orthopolyex;
logy=log(concentration);
run;

proc gplot data=orthopolyex;
plot logy*time;
run;
```

![Graph showing log concentration against time]
1-way ANOVA with orthogonal polynomial contrasts

```
proc glm data=orthopolyex plot=diagnostics;
  class time;
  model logy=time;
  contrast 'linear contrast' time -2 -1 0 1 2;
  contrast 'quadratic contrast' time 2 -1 -2 -1 2;
  contrast 'cubic contrast' time -1 2 0 -2 1;
  contrast 'quartic contrast' time 1 -4 6 -4 1;
run;
```

The GLM Procedure

Dependent Variable: logy

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4</td>
<td>24.35092977</td>
<td>6.08773244</td>
<td>539.43</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>0.11285518</td>
<td>0.01128552</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>14</td>
<td>24.46378495</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>4</td>
<td>24.35092977</td>
<td>6.08773244</td>
<td>539.43</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>4</td>
<td>24.35092977</td>
<td>6.08773244</td>
<td>539.43</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contrast</th>
<th>DF</th>
<th>Contrast SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear contrast</td>
<td>1</td>
<td>24.29199104</td>
<td>24.29199104</td>
<td>2152.49</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>quadratic contrast</td>
<td>1</td>
<td>0.00386341</td>
<td>0.00386341</td>
<td>0.34</td>
<td>0.5715</td>
</tr>
<tr>
<td>cubic contrast</td>
<td>1</td>
<td>0.00618826</td>
<td>0.00618826</td>
<td>0.55</td>
<td>0.4760</td>
</tr>
<tr>
<td>quartic contrast</td>
<td>1</td>
<td>0.04888706</td>
<td>0.04888706</td>
<td>4.33</td>
<td>0.0641</td>
</tr>
</tbody>
</table>

SAS code for regression model with quartic polynomial (Type I SS relevant)

```
data orthopolyex; set orthopolyex;
time2=time*time;
time3=time*time2;
time4=time*time3;
run;
```

```
proc glm data=orthopolyex;
model logy=time time2 time3 time4;
run;
```
The GLM Procedure (regression model output)

Dependent Variable: logy

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Source DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4</td>
<td>4</td>
<td>24.29199104</td>
<td>24.29199104</td>
<td>2152.49</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>10</td>
<td>0.00386341</td>
<td>0.00386341</td>
<td>0.34</td>
<td>0.5715</td>
</tr>
<tr>
<td>Corrected Total</td>
<td>14</td>
<td>14</td>
<td>0.00618826</td>
<td>0.00618826</td>
<td>0.55</td>
<td>0.4760</td>
</tr>
<tr>
<td>Corrected Total</td>
<td>14</td>
<td>14</td>
<td>0.04888706</td>
<td>0.04888706</td>
<td>4.33</td>
<td>0.0641</td>
</tr>
</tbody>
</table>

Notice that the four $SS_{contrast}$ values from the 1-way ANOVA match the Type I SS (i.e. sequential sums of squares) from the regression model with a quartic polynomial.

While this set of orthogonal polynomial contrasts is nice to know and can be very useful at times, the availability of computing has reduced our dependency on polynomial contrasts and the frequency of their use.