Statistical Consulting Topics

SAS mixed models:

- Split-plot with blocking at the whole-plot level

- Steel is normalized by heating above the critical temperature, soaking, and then air cooling. This process increases the strength of the steel, refines the grain, and homogenizes the structure.


- An experiment is performed to determine the effect of temperature and heat treatment time on the strength of the normalized steel.
  - Two temperatures and three times are considered.
  - One full replicate of the experiment is performed during a shift by heating the oven
to a randomly selected temperature and inserting three specimens. After 10 minutes, one specimen is removed. After 20 minutes, the second is removed. After 30 minutes, the third is removed. Then, during the same shift, the temperature is changed to the other level, and the process is repeated.

— Data was collected over four shifts.

• In order to utilize the full capacity of the oven, it makes the most sense to randomly choose a *Temp* assignment for the oven, then place 3 specimens in the oven, one for each level of *Time*. This essentially gives us 1 *Temp* observation, and 3 *Time* observations at the given *Temp*. 
• A CRD would require us to put only 1 specimen per oven at a given Temperature/Time combination... rather wasteful. Plus, this way, we’ll have a very good comparison across times for the 3 specimens in the same oven run because we’ve controlled for oven run-to-oven run variability. This is an example of when your resources naturally lead you to a design other than a CRD.

• The random blocking factor in this experiment is Shift and the ‘whole-plot treatment’ is Temp. Shift is crossed with Temp as every shift observes every temperature level.

• One way to show this diagram:

\{\text{SHIFT}_R \times \text{temperature}\}_R \times \text{time}
Below, I’ve used the traditional choice for this type of split-plot and put both the SHIFT*time term and SHIFT*time*temp term into the split-plot error (or residual), but it is also possible to include the SHIFT*time term into the model (but for this assignment, use this ANOVA below).

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SHIFT</strong></td>
<td>3</td>
</tr>
<tr>
<td>temperature</td>
<td>1</td>
</tr>
<tr>
<td><strong>SHIFT</strong>*temperature</td>
<td>3</td>
</tr>
<tr>
<td>time</td>
<td>2</td>
</tr>
<tr>
<td>time*temperature</td>
<td>2</td>
</tr>
<tr>
<td><strong>RESIDUAL</strong></td>
<td>12</td>
</tr>
<tr>
<td>c. total</td>
<td>23</td>
</tr>
</tbody>
</table>

contains both

\[ \text{SHIFT} \times \text{time} \]
\[ \text{SHIFT} \times \text{temp} \times \text{time} \]
SAS code:

```
pic mixed data=steel
  plots(only)=residualpanel(conditional);
  class temperature time shift;
  model strength=temperature|time/ddfm=satterth residual;
  random shift shift*temperature;
run;
```

SAS output:

The Mixed Procedure

Model Information

Data Set WORK.STEEL
Dependent Variable Strength
Covariance Structure Variance Components
Estimation Method REML
Residual Variance Method Profile
Fixed Effects SE Method Model-Based
Degrees of Freedom Method Satterthwaite

Class Level Information

<table>
<thead>
<tr>
<th>Class</th>
<th>Levels</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>2</td>
<td>1500 1600</td>
</tr>
<tr>
<td>Time</td>
<td>3</td>
<td>10 20 30</td>
</tr>
<tr>
<td>Shift</td>
<td>4</td>
<td>1 2 3 4</td>
</tr>
</tbody>
</table>
Dimensions

Covariance Parameters 3
Columns in X 12
Columns in Z 12
Subjects 1
Max Obs Per Subject 24

Covariance Parameter Estimates

Cov Parm            Estimate
Shift               28.7222
Temperature*Shift   6.6389
Residual            60.2361

Type 3 Tests of Fixed Effects

<table>
<thead>
<tr>
<th>Effect</th>
<th>Num DF</th>
<th>Den DF</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>1</td>
<td>3</td>
<td>29.20</td>
<td>0.0124</td>
</tr>
<tr>
<td>Time</td>
<td>2</td>
<td>12</td>
<td>1.32</td>
<td>0.3028</td>
</tr>
<tr>
<td>Temperature*Time</td>
<td>2</td>
<td>12</td>
<td>6.60</td>
<td>0.0117</td>
</tr>
</tbody>
</table>

Because there is significant interaction, we will consider the test for a temperature effect at each time point.
SAS code:

```sas
proc mixed data=steel
    plots(only)=residualpanel(conditional);
class temperature time shift;
model strength=temperature|time/ddfm=satterth residual;
random shift shift*temperature;
lsmeans temperature*time/slice=time;
run;
```

SAS output:

Tests of Effect Slices

<table>
<thead>
<tr>
<th>Effect</th>
<th>Num</th>
<th>Den</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature*Time</td>
<td>10</td>
<td>1</td>
<td>24.72</td>
<td>0.0003</td>
</tr>
<tr>
<td>Temperature*Time</td>
<td>20</td>
<td>1</td>
<td>21.80</td>
<td>0.0005</td>
</tr>
<tr>
<td>Temperature*Time</td>
<td>30</td>
<td>1</td>
<td>0.37</td>
<td>0.5563</td>
</tr>
</tbody>
</table>

Temp-by-time +/- 1 SE mean

Significant temperature effect at time 10 & 20 but not time 30
Conditional Residuals for Strength

Residual Statistics
- Observations: 24
- Minimum: -9.673
- Mean: 2.414
- Maximum: 15.263
- Std Dev: 6.2615

Fit Statistics
- Objective: 138.32
- AIC: 144.32
- AICc: 146.04
- BIC: 142.48