Homework 5 **Name:** ______________________________

Elementary Statistics & Inference (STAT:1020; Bognar)

Print the pdf file, show your work in the provided space, scan pages (in order) into a single pdf file, submit in Gradescope. You may use an iPad.

1. Textbook 13.53

   (a)

   (b)

2. The probability that a passenger will attempt to board an airplane with illegal drugs is 0.005 (i.e. $P(D) = 0.005$). Given that a passenger has illegal drugs, the probability that the alarm will sound is 0.97 (i.e. $P(A|D) = 0.97$). If a passenger does not have illegal drugs, the probability that the alarm will not sound is 0.95 (i.e. $P(A^c|D^c) = 0.95$).

   (a) What is the sensitivity of the drug detection machine?

   (b) What is the specificity of the drug detection machine?

   (c) Find the probability that the alarm does not sound given that the passenger is carrying drugs, i.e. find $P(A^c|D)$.

(d) Suppose a passenger is randomly selected. Use the law of total probability to find the probability that the alarm sounds when he/she enters security (i.e. find $P(A)$). *Hint: D and D^c form a partition.*

(e) Given that the alarm sounds, find the probability that the passenger actually has illegal drugs, i.e. find $P(D|A)$. This quantity is known as the "predictive value of a positive test".

(f) Find the "predictive value of a negative test" (i.e. find $P(D^c|A^c)$). In words, what does this quantity mean?

3. A farm has two types of trees: 30% are orange trees (O) and 70% are apple trees (A). Frost (F) has damaged 40% of the orange trees (i.e. $P(F|O) = 0.40$) and 10% of the apple trees.

   (a) Find the probability that a randomly selected tree was damaged by frost *and* is an apple tree.

   (b) Use the law of total probability to find the probability that a randomly selected tree has been damaged by frost, i.e. find $P(F)$. *Hint: O and A form a partition.*

(c) Given that a randomly selected tree has been damaged by frost, determine the probability that it is an apple tree, i.e. find $P(A|F)$.

4. A basket contains 4 puppies: one of the puppies has 1 spot, one of the puppies has 2 spots, and the remaining two puppies have 4 spots. Suppose *two* puppies are selected at random *without* replacement. Let the random variable X equal the *total* number of spots on the selected puppies.

   (a) Find the probability distribution (probability mass function) of X.

   (b) Find the probability that the puppies have a total of 5 spots, i.e. find $P(X = 5)$.

   (c) Find the probability that the puppies have a total of 6 or more spots, i.e. find $P(X \geq 6)$.

   (d) On average, how many spots do we expect on the two selected puppies? In other words, find $\mu = E(X)$.

   (e) Compute $\sigma^2 = Var(X)$.

5. A street vendor is asking people to play a simple game. You roll a pair of dice. If the sum on the dice is 10 or higher, you win \$10. If you roll a pair of 1's, you win \$50. Otherwise you lose \$5. If the random variable X equals your win (or loss) for each play, find $\mu = E(X)$ (i.e. figure out how much we expect to win or lose for each play, on average). Is it wise to play this game? Why? *Hint: First find the probability distribution (probability mass function) of* X.

6. Suppose a bowl has 5 chips; two chips are labeled "2", and three chips are labeled "3". Suppose *two* chips are selected at random *with* replacement. Let the random variable X equal the *product* of the two draws (e.g. if the first draw is a 2 (2_1) and the second draw is a 3 (3_2), then the product is $2 \times 3 = 6$).

   (a) Find the probability distribution (probability mass function) of X.

   (b) Find the probability that the *product* of the two draws is less than or equal to 6, i.e. find $P(X \leq 6)$.

   (c) Compute the expected value of X, $\mu = E(X)$.

   (d) Compute $\sigma = SD(X)$.