Homework 10 **Name:** ______________________________

Prob. and Stat. for Eng. (STAT:2020; Bognar)

Print this pdf file, show your work in the provided space, use scanning app to scan pages (in order) into a single pdf file, submit in Gradescope. Be sure to get entire page in each shot — lay each page flat when scanning. You can use an iPad/tablet too. The Gradescope app works well for submitting too. Make sure the pages upload in order.

1. A physicist needs to estimate the density of a cube (all sides of the cube have equal length). Density (in kg/m^3) can be found by

$$D = U(M, L) = \frac{M}{L^3}$$

where M is the mass of the object (in kg) and L is the length of a side of the cube (in meters). Assume the mass $M \sim \cdot(\mu_M, \sigma_M^2 = 0.02^2)$ and assume the length $L \sim \cdot(\mu_L, \sigma_L^2 = 0.005^2)$. Assume M and L are independent. The physicist measured the mass and length and found $m = 1.0$ and $l = 0.1$ Ohms.

   (a) Approximate μ_U.

   (b) Approximate σ_U.

   (c) Write the estimate of the density, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

2. An engineer needs to estimate the amount of power dissipated by a wire-wound resistor. Power (in watts) can be found by

$$P = U(V, R) = \frac{V^2}{R}$$

where V is the voltage (in volts) and R is the resistance (in Ohms). In this particular application, assume the voltage $V \sim \cdot(\mu_V, \sigma_V^2 = 0.2^2)$ and assume the resistance $R \sim \cdot(\mu_R, \sigma_R^2 = 0.1^2)$. Assume V and R are independent. The engineer measured the voltage and resistance and found $v = 14.4$ volts and $r = 8.2$ Ohms.

   (a) Approximate μ_U.

(b) Approximate σ_U.

(c) Write the estimate of the power, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

3. Textbook 8.24

4. Textbook 8.25

   (a)

   (b)

5. Textbook 8.26

(a)

(b)

(c)

6. A bowl contains 3 chips: the chips labeled 0, 2, and 4. A chip is randomly selected from the bowl. Let X denote the number printed on the chip. The probability mass function (probability distribution) of X is

$x:$	0	2	4
$P(X=x):$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

(a) Find the mean of X, i.e. find $\mu = E(X) = \sum_x xP(X=x)$.

(b) Find the standard deviation of X, i.e. find $\sigma = SD(X) = \sqrt{\sum_x (x-\mu)^2 P(X=x)}$.

(c) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find the sampling distribution of $\bar{X}$.

(d) Determine the mean of $\bar{X}$, i.e. compute $\mu_{\bar{X}} = E(\bar{X}) = \sum_{\bar{x}} \bar{x} P(\bar{X} = \bar{x})$ using your answer in part (6c).

(e) According to the theorem given in class, the mean of $\bar{X}$ is $\mu_{\bar{X}} = E(\bar{X}) = \mu$. Does this hold true when you compare parts (6d) and (6a)?

(f) Determine the standard deviation of $\bar{X}$, i.e. compute $\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{\sum_{\bar{x}} \left(\bar{x} - \mu_{\bar{X}}\right)^2 P(\bar{X} = \bar{x})}$ using your answer in part (6c).

(g) According to the theorem given in class, the standard deviation of $\bar{X}$ is $\sigma_{\bar{X}} = SD(\bar{X}) = \sigma/\sqrt{n}$. Compute $\sigma/\sqrt{n}$ (remember, we derived σ in part (6b)). Does this equal the result from part (6f)?

(h) Suppose 100 chips are randomly selected from the bowl with replacement. Let $\bar{X}$ denote the mean of the selected chips. Find what is the approximate distribution of $\bar{X}$?

(i) Suppose 100 chips are randomly selected from the bowl with replacement. Let $\bar{X}$ denote the mean of the selected chips. Approximate the probability that $\bar{X}$ is between 2.1 and 2.4.