Homework 10 **Name:** ______________________________

Prob. and Stat. for Eng. (STAT:2020; Bognar)

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1. Textbook 8.24

2. Textbook 8.25

   (a)

   (b)

3. Textbook 8.26

   (a)

(b)

(c)

4. A bowl contains 3 chips: the chips labeled 0, 2, and 4. A chip is randomly selected from the bowl. Let X denote the number printed on the chip. The probability mass function (probability distribution) of X is

$$\begin{array}{rccc} x: & 0 & 2 & 4 \\ P(X=x): & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}$$

(a) Find the mean of X, i.e. find $\mu = E(X) = \sum_x xP(X = x)$.

(b) Find the standard deviation of X, i.e. find $\sigma = SD(X) = \sqrt{\sum_x (x-\mu)^2 P(X = x)}$.

(c) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find the sampling distribution of $\bar{X}$.

(d) Suppose 2 chips are randomly selected from the bowl *with* replacement. Find $P(\bar{X} \le 1)$.

(e) Determine the mean of $\bar{X}$, i.e. compute $\mu_{\bar{X}} = E(\bar{X}) = \sum_{\bar{x}} \bar{x} P(\bar{X} = \bar{x})$ using your answer in part (4c).

(f) According to the theorem given in class, the mean of $\bar{X}$ is $\mu_{\bar{X}} = E(\bar{X}) = \mu$. Does this hold true when you compare parts (4e) and (4a)?

(g) Determine the standard deviation of $\bar{X}$, i.e. compute $\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{\sum_{\bar{x}} (\bar{x} - \mu_{\bar{X}})^2 P(\bar{X} = \bar{x})}$ using your answer in part (4c).

(h) According to the theorem given in class, the standard deviation of $\bar{X}$ is $\sigma_{\bar{X}} = SD(\bar{X}) = \sigma/\sqrt{n}$. Compute $\sigma/\sqrt{n}$ (remember, we derived σ in part (4b)). Does this equal the result from part (4g)?

(i) Suppose 100 chips are randomly selected from the bowl with replacement. Let $\bar{X}$ denote the mean of the selected chips. Find what is the approximate distribution of $\bar{X}$?

(j) Suppose 100 chips are randomly selected from the bowl with replacement. Let $\bar{X}$ denote the mean of the selected chips. Approximate the probability that $\bar{X}$ is between 2.1 and 2.4.

5. A physicist needs to estimate the density of a cube (all sides of the cube have equal length). Density (in kg/m^3) can be found by

$$D = U(M, L) = \frac{M}{L^3}$$

where M is the mass of the object (in kg) and L is the length of a side of the cube (in meters). Assume the mass $M \sim \cdot(\mu_M, \sigma_M^2 = 0.02^2)$ and assume the length $L \sim \cdot(\mu_L, \sigma_L^2 = 0.005^2)$. Assume M and L are independent. The physicist measured the mass and length and found $m = 1.0$ and $l = 0.1$ Ohms.

(a) Approximate μ_D.

(b) Approximate σ_D.

(c) Write the estimate of the density, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

6. An engineer needs to estimate the amount of power dissipated by a wire-wound resistor. Power (in watts) can be found by

$$P = U(V, R) = \frac{V^2}{R}$$

where V is the voltage (in volts) and R is the resistance (in Ohms). In this particular application, assume the voltage $V \sim \cdot(\mu_V, \sigma_V^2 = 0.2^2)$ and assume the resistance $R \sim \cdot(\mu_R, \sigma_R^2 = 0.1^2)$. Assume V and R are independent. The engineer measured the voltage and resistance and found $v = 14.4$ volts and $r = 8.2$ Ohms.

(a) Approximate μ_P.

(b) Approximate σ_P.

(c) Write the estimate of the power, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.