HOMEWORK 13 **NAME:** ______________________________

PROB. AND STAT. FOR ENG. (STAT:2020; BOGNAR)

Print this pdf file (do not use notebook paper), show your work in the provided space, use scanning app to scan pages (in order) into a single pdf file, submit in Gradescope. Be sure to get entire page in each shot — lay each page flat when scanning. You can use an iPad/tablet too. The Gradescope app works well for submitting too. Make sure the pages upload in order.

1. The time, X (in minutes), it takes a robot to weld steel beams can be modeled by a $X \sim N(\mu, \sigma^2)$ distribution. A worker observed the welding times for 18 randomly selected beams and found the sample mean welding time was $\bar{x} = 140.2$ minutes with sample standard deviation $s = 8.5$ minutes.

   (a) Test $H_0 : \mu = 145$ versus $H_a : \mu \neq 145$ at the $\alpha = 0.01$ significance level using a 3-step test.

   (b) Based upon your answer in part (a), does μ significantly differ from 145? Why?

   (c) Approximate the $p-$value for the test in part (a).

   (d) Based upon your answer in part (c), does μ significantly differ from 145? Why?

   (e) Find a 99% confidence interval for μ.

   (f) Based upon your answer in part (e), does μ significantly differ from 145? Why?

   (g) If the times were not normally distributed, could we still do inference for μ? Why?

2. The time, X (in minutes), it takes a person to weld steel beams can be modeled by a $X \sim N(\mu, \sigma^2)$ distribution. A worker was timed when welding 16 randomly selected beams; the sample mean welding time was $\bar{x} = 175.4$ minutes with sample standard deviation $s = 18.1$ minutes.

   (a) Test $H_0 : \mu = 160$ versus $H_a : \mu > 160$ at the $\alpha = 0.05$ significance level using a 3-step test.

   (b) Based upon your answer in part (a), is μ significantly greater than 160? Why?

   (c) Approximate the $p-$value for the test in part (a).

   (d) Based upon your answer in part (c), is μ significantly greater than 160? Why?

   (e) Find a 95% one-sided lower bound CI for μ.

3. A sociologist collected a random sample of 13 statistics majors and 14 sociology majors. The students were asked about how many hours per week they spend socializing. The results are summarized in the following table. Assume that the amount of socialization for statistics majors X_1 follows a normal distribution with mean μ_1 and standard deviation σ_1 (i.e. $X_1 \sim N(\mu_1, \sigma_1)$), while the amount of socialization for sociology majors X_2 follows a normal distribution with mean μ_2 and standard deviation σ_2 (i.e. $X_2 \sim N(\mu_2, \sigma_2)$). Because the sample standard deviations s_1 and s_2 are quite similar, lets make the reasonable assumption that $\sigma_1 = \sigma_2$.

Statistics:	$n_1 = 13$	$\bar{x}_1 = 32.1$	$s_1 = 10$
Sociology:	$n_2 = 14$	$\bar{x}_2 = 23.0$	$s_2 = 12$

   (a) Find a 95% confidence interval for $\mu_1 - \mu_2$.

(b) Based upon your answer in (3a), is there a significant difference in the mean time spent socializing between statistics and sociology majors? Why?

(c) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Based upon your answer in (3a), will H_0 be rejected? Why?

(d) Based upon your answer in (3c), will the $p-$value be less than 0.05 or greater than 0.05? Why?

(e) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(f) Based upon your answer in (3e), is there a significant difference between the average socialization times? Why?

(g) Approximate the $p-$value for the test in (3e) using the $t-$table.

(h) Use the $t-$Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the $p-$value for the test in (3e).

(i) Consider the test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.01$ significance level. Based upon your answer in (3g) and (3h), do you reject H_0? Why?

(j) Could we do the above analysis if the study times were *not* normally distributed? Why?

4. The level of iodine in Company A's table salt follows a $N(\mu_1, \sigma_1)$ distribution, while the level in Company B's salt follows a $N(\mu_2, \sigma_2)$ distribution. A random sample of each companies' product yielded

Company A:	$n_1 = 16$	$\bar{x}_1 = 22.4$	$s_1 = 1.0$
Company B:	$n_2 = 9$	$\bar{x}_2 = 27.5$	$s_2 = 1.4$

Assume that $\sigma_1 = \sigma_2$.

(a) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 < \mu_2$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in (4a), is the mean iodine level for Company A significantly lower than Company B? Why?

(c) Approximate the $p-$value for the test in (4a) using the $t-$table.

(d) Based upon your answer in (4c), does it look like the mean iodine level for Company A significantly lower than Company B? Why?