**Homework 9** **Name:** ______________________________

**Prob. and Stat. for Eng. (STAT:2020; Bognar)**

*Print this pdf file, show your work in the provided space, use scanning app to scan pages (in order) into a single pdf file, submit in Gradescope. Be sure to get entire page in each shot — lay each page flat when scanning. You can use an iPad/tablet too. The Gradescope app works well for submitting too. Make sure the pages upload in order.*

1. Suppose $X_1$, $X_2$, and $X_3$ are independent random variables where

$$X_1 \sim N(\mu_1 = 1, \sigma_1^2 = 1^2) \qquad X_2 \sim N(\mu_2 = 2, \sigma_2^2 = 2^2) \qquad X_3 \sim N(\mu_3 = 3, \sigma_3^2 = 3^2)$$

   (a) Let $W = X_1 - X_2$. What is the distribution of $W$? Be sure to state all parameters.

   (b) Using your answer in (1a), find $P(X_1 > X_2)$.

   (c) Let $W = X_1 - 6X_2 + 2X_3$. What is the distribution of $W$? Be sure to state all parameters.

   (d) Using your answer in (1c), find $P(X_1 - 6X_2 > 5 - 2X_3)$.

2. An artist makes pottery. There are two major steps: wheel throwing and firing. The time (in minutes) for wheel throwing can be modeled by a $X_1 \sim N(\mu = 40, \sigma^2 = 2^2)$ distribution and the time for firing can be modeled by a $X_2 \sim N(\mu = 60, \sigma^2 = 3^2)$ distribution. Assume independence.

   (a) Determine the probability that a piece of pottery will take longer than 110 minutes.

(b) Suppose 10 pieces of pottery are randomly selected. Determine the probability that the mean firing time $\bar{X}$ is between 58 and 61 minutes.

(c) Suppose 10 pieces of pottery are randomly selected. Let $\bar{X}$ denote the sample mean firing time. Determine the 10th percentile of $\bar{X}$.

(d) Determine the probability that 2 pieces of pottery will take less than 210 minutes using a linear combination. *Think carefully when doing this problem. Note that we can not simply find* $P(2X_1 + 2X_2 < 210)$. *Hint: Let* $Y_1$ *denote the completion time for the first piece, and let* $Y_2$ *denote the completion time for the second piece.*

3. Suppose $X_1, \ldots, X_{25}$ are independent and identically distributed normal random variables with mean $\mu = 100$ and standard deviation $\sigma = 20$, i.e.

$$X_i \overset{iid}{\sim} N(\mu = 100, \sigma^2 = 20^2 = 400)$$

for $i = 1, \ldots, 25$. Let the sample mean $\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i$.

(a) Find the distribution of the sample mean $\bar{X}$.

(b) Find $P(98 < \bar{X} < 105)$.

(c) Find the 20th percentile of $\bar{X}$.

4. Suppose $X_1$, $X_2$, and $X_3$ are independent random variables where

$$X_1 \sim N(\mu_1 = \mu, \sigma_1^2 = 1^2) \qquad X_2 \sim N(\mu_2 = \mu, \sigma_2^2 = 2^2) \qquad X_3 \sim N(\mu_3 = \mu, \sigma_3^2 = 3^2)$$

If $0.9 = P(5X_1 + 2X_2 - 4X_3 > 10)$, find $\mu$.

5. Suppose a farmer has a crop circle. The area (in square meters) of the crop circle is

$$A = U(R) = \pi R^2$$

where the radius $R \sim \cdot(\mu_R = 100, \sigma_R^2 = 4^2)$ (i.e. $100 \pm 4$) meters.

(a) Approximate $\mu_A$.

(b) Approximate $\sigma_A$.

(c) Write the estimate of the area, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

6. A sewage treatment facility has a large circular holding tank. The tank is known to be 3.2 meters high. A worker wishes to measure the volume of the tank (in cubic meters). The volume can be found by

$$V = U(C) = \frac{3.2C^2}{4\pi}$$

where $C$ is the circumference of the tank (in meters). The large circumference, however, is very difficult to measure accurately due to the limited measuring equipment available. Assume $C \sim \cdot(\mu_C, \sigma_C^2 = 40^2)$ meters. The worker approximated the circumference $C$ to be $c = 210$ meters (i.e. $210 \pm 40$).

(a) Approximate $\mu_V$.

(b) Approximate $\sigma_V$.

(c) Write the estimate of the volume, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.