HOMEWORK 9 PROB. AND STAT. FOR ENG. (STAT:2020; BOGNAR)

NAME:

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1. Suppose X_1, X_2 , and X_3 are independent random variables where

$$X_1 \sim N(\mu_1 = 1, \sigma_1^2 = 1^2)$$
 $X_2 \sim N(\mu_2 = 2, \sigma_2^2 = 2^2)$ $X_3 \sim N(\mu_3 = 3, \sigma_3^2 = 3^2)$

(a) Let $W = X_1 - X_2$. What is the distribution of W? Be sure to state all parameters.

(b) Using your answer in (1a), find $P(X_1 > X_2)$.

(c) Let $W = X_1 - 6X_2 + 2X_3$. What is the distribution of W? Be sure to state all parameters.

(d) Using your answer in (1c), find $P(X_1 - 6X_2 > 5 - 2X_3)$.

- 2. An artist makes pottery. There are two major steps: wheel throwing and firing. The time (in minutes) for wheel throwing can be modeled by a $X_1 \sim N(\mu = 40, \sigma^2 = 2^2)$ distribution and the time for firing can be modeled by a $X_2 \sim N(\mu = 60, \sigma^2 = 3^2)$ distribution. Assume independence.
 - (a) Determine the probability that a piece of pottery will take longer than 110 minutes.

(b) Suppose 10 pieces of pottery are randomly selected. Determine the probability that the mean firing time \bar{X} is between 58 and 61 minutes.

(c) Suppose 10 pieces of pottery are randomly selected. Let \bar{X} denote the sample mean firing time. Determine the 10th percentile of \bar{X} .

(d) Determine the probability that 2 pieces of pottery will take less than 210 minutes using a linear combination. Think carefully when doing this problem. Note that we can not simply find $P(2X_1 + 2X_2 < 210)$. Hint: Let Y_1 denote the completion time for the first piece, and let Y_2 denote the completion time for the second piece.

3. Suppose X_1, \ldots, X_{25} are independent and identically distributed normal random variables with mean $\mu = 100$ and standard deviation $\sigma = 20$, i.e.

$$X_i \stackrel{iid}{\sim} N(\mu = 100, \sigma^2 = 20^2 = 400)$$

for i = 1, ..., 25. Let the sample mean $\bar{X} = \frac{1}{25} \sum_{i=1}^{25} X_i$.

(a) Find the distribution of the sample mean \bar{X} .

(b) Find $P(98 < \bar{X} < 105)$.

(c) Find the 20th percentile of \bar{X} .

4. Suppose X_1, X_2 , and X_3 are independent random variables where

$$X_1 \sim N(\mu_1 = \mu, \sigma_1^2 = 1^2) \qquad X_2 \sim N(\mu_2 = \mu, \sigma_2^2 = 2^2) \qquad X_3 \sim N(\mu_3 = \mu, \sigma_3^2 = 3^2)$$

If $0.9 = P(5X_1 + 2X_2 - 4X_3 > 10)$, find μ .

5. Suppose a farmer has a crop circle. The area (in square meters) of the crop circle is

$$A = U(R) = \pi R^2$$

where the radius $R \sim \cdot (\mu_R = 100, \sigma_R^2 = 4^2)$ (i.e. 100 ± 4) meters.

(a) Approximate μ_A .

(b) Approximate σ_A .

- (c) Write the estimate of the area, along with the estimated error, in engineering (i.e. \pm) notation. Be sure to state the units.
- 6. A sewage treatment facility has a large circular holding tank. A worker wishes to measure the volume of the tank (in cubic meters). The volume can be found by

$$V = U(C) = \frac{C^2 h}{4\pi}$$

where h is the height of the tank (in meters), and C is the circumference of the tank (in meters). The height h can be measured without error. The large circumference, however, is very difficult to measure accurately due to the limited measuring equipment available. Assume $C \sim (\mu_C, \sigma_C^2 = 40^2)$ meters. The worker measured the height to be h = 3.2 meters and the circumference C to be c = 210 meters.

(a) Approximate μ_V .

(b) Approximate σ_V .

(c) Write the estimate of the volume, along with the estimated error, in engineering (i.e. \pm) notation. Be sure to state the units.