Homework (Propagation of Error, a.k.a. Delta Method)
Prob. and Stat. for Eng. (STAT:2020; Bognar)

1. Suppose a farmer has a crop circle. The area (in square meters) of the crop circle is

$$A = U(R) = \pi R^2$$

where the radius $R \sim \cdot(\mu_R = 100, \sigma_R^2 = 4^2)$ (i.e. 100 ± 4) meters.

   (a) Approximate μ_A.

   (b) Approximate σ_A.

   (c) Write the estimate of the area, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

2. A sewage treatment facility has a large circular holding tank. A worker wishes to measure the volume of the tank (in cubic meters). The volume can be found by

$$V = U(C) = \frac{C^2 h}{4\pi}$$

where h is the height of the tank (in meters), and C is the circumference of the tank (in meters). The height h can be measured without error. The large circumference, however, is very difficult to measure accurately due to the limited measuring equipment available. Assume $C \sim \cdot(\mu_C, \sigma_C^2 = 40^2)$ meters. The worker measured the height to be $h = 3.2$ meters and the circumference C to be $c = 210$ meters.

   (a) Approximate μ_V.

   (b) Approximate σ_V.

   (c) Write the estimate of the volume, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

3. A physicist needs to estimate the density of a cube (all sides of the cube have equal length). Density (in kg/m^3) can be found by

$$D = U(M, L) = \frac{M}{L^3}$$

where M is the mass of the object (in kg) and L is the length of a side of the cube (in meters). Assume the mass $M \sim \cdot(\mu_M, \sigma_M^2 = 0.02^2)$ and assume the length $L \sim \cdot(\mu_L, \sigma_L^2 = 0.005^2)$. Assume M and L are independent. The physicist measured the mass and length and found $m = 1.0$ and $l = 0.1$ Ohms.

   (a) Approximate μ_D.

   (b) Approximate σ_D.

   (c) Write the estimate of the density, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.

4. An engineer needs to estimate the amount of power dissipated by a wire-wound resistor. Power (in watts) can be found by

$$P = U(V, R) = \frac{V^2}{R}$$

where V is the voltage (in volts) and R is the resistance (in Ohms). In this particular application, assume the voltage $V \sim \cdot(\mu_V, \sigma_V^2 = 0.2^2)$ and assume the resistance $R \sim \cdot(\mu_R, \sigma_R^2 = 0.1^2)$. Assume V and R are independent. The engineer measured the voltage and resistance and found $v = 14.4$ volts and $r = 8.2$ Ohms.

   (a) Approximate μ_P.

   (b) Approximate σ_P.

   (c) Write the estimate of the power, along with the estimated error, in engineering (i.e. $\pm$) notation. Be sure to state the units.