Homework (Bognar)
Introduction to Mathematical Statistics I (STAT:3100)

1. For problem 4.3-1 in the textbook, answer the following questions.
   (a) Find $P(X = 1, Y \leq 3)$.
   (b) Find the marginal pmf of X, $f_X(x)$. *Be sure to state the support.*
   (c) Find $\mu_X = E(X)$.
   (d) Find $\sigma_X^2 = Var(X)$.
   (e) Find the marginal pmf of Y, $f_Y(y)$. *Be sure to state the support.*
   (f) Find $P(Y \geq 3)$.
   (g) Are X and Y independent? Why?
   (h) Find $P(Y = X^2)$.
   (i) Find the conditional pmf of Y given $X = x$, $f_{Y|X=x}(y)$. *Be sure to state the support.*
   (j) Find the expected value of Y given $X = x$, $E(Y|X = x)$.
   (k) Find the expected value of Y given $X = 1$, $E(Y|X = 1)$.
   (l) Find $P(Y \geq 3|X = 1)$.
   (m) Find $P(X = 1|Y = 2)$.

2. Suppose the continuous random variables X and Y have joint pdf

$$f_{XY}(x, y) = \begin{cases} x + y & 0 < x < 1 \quad 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $P(X > 0.9, Y < 0.5)$.
   (b) Find $E(XY)$.
   (c) Find the marginal pdf of X, $f_X(x)$. *Be sure to state the support.*
   (d) Find $\mu_X = E(X)$.
   (e) Find $\sigma_X^2 = Var(X)$.
   (f) Find the marginal pdf of Y, $f_Y(y)$. *Be sure to state the support.*
   (g) Find $P(Y > 0.5)$.
   (h) Are X and Y independent? Why?
   (i) Find $P(Y > X^2)$.
   (j) Find the conditional pdf of X given $Y = y$, $f_{X|Y=y}(x)$. *Be sure to state the support.*
   (k) Find the expected value of X given $Y = y$, $E(X|Y = y)$.
   (l) Find the expected value of X given $Y = 0.2$, $E(X|Y = 0.2)$.
   (m) Find $P(X < 0.2|Y = 0.9)$.
   (n) Find $P(Y < 0.1|X = 0.5)$.