

HOMEWORK (BOGNAR)
INTRODUCTION TO MATHEMATICAL STATISTICS I (STAT:3100)

1. Suppose the continuous random variables X_1 and X_2 have joint pdf

$$f_{X_1 X_2}(x_1, x_2) = \begin{cases} 2 & 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y_1 = X_1 + X_2$ and $Y_2 = X_2$.

- (a) Use the change of variable (Jacobian) technique to find the joint pdf of Y_1 and Y_2 , $f_{Y_1 Y_2}(y_1, y_2)$. *Be sure to state the support.*
(b) Find the marginal pdf of Y_2 , $f_{Y_2}(y_2)$. *Be sure to state the support.*
(c) Find the marginal pdf of Y_1 , $f_{Y_1}(y_1)$. *Note that this marginal pdf will need to be defined piecewise, i.e.*

$$f_{Y_1}(y_1) = \begin{cases} \ominus & 0 < y_1 < 1 \\ \ominus & 1 \leq y_1 < 2 \\ 0 & \text{otherwise} \end{cases}$$

Note that Y_1 has a “triangular distribution”.

- (d) Find $E(X_1 + X_2) = E(Y_1)$.

2. Suppose the continuous random variables X_1 and X_2 have joint pdf

$$f_{X_1 X_2}(x_1, x_2) = \begin{cases} 2x_1 & 0 < x_1 < 1, \ 0 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y_1 = X_1/X_2$ and $Y_2 = X_2$.

- (a) Use the change of variable (Jacobian) technique to find the joint pdf of Y_1 and Y_2 , $f_{Y_1 Y_2}(y_1, y_2)$. *Be sure to state the support.*
(b) Find the marginal pdf of Y_2 , $f_{Y_2}(y_2)$. *Be sure to state the support.*
(c) Find the marginal pdf of Y_1 , $f_{Y_1}(y_1)$. *Note that this marginal pdf will need to be defined piecewise, i.e.*

$$f_{Y_1}(y_1) = \begin{cases} \ominus & 0 < y_1 < 1 \\ \ominus & 1 \leq y_1 < \infty \\ 0 & \text{otherwise} \end{cases}$$

- (d) Find $P(X_1/X_2 < 3) = P(Y_1 < 3)$.