Homework 10 **Name:** ______________________

Biostatistics (STAT:3510; Bognar)

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1. The amount of socialization (in hours per week) for statistics majors can be modeled by a $X_1 \sim N(\mu_1, \sigma_1)$ distribution, while the amount of socialization for sociology majors can be modeled by a $X_2 \sim N(\mu_2, \sigma_2)$ distribution. A sociologist collected a random sample of 13 statistics majors and 14 sociology majors; the socialization results are below. Because the sample standard deviations s_1 and s_2 are quite similar, lets make the reasonable assumption that $\sigma_1 = \sigma_2$.

$$\text{Statistics} : n_1 = 13,\ \bar{x}_1 = 32.1,\ s_1 = 10 \qquad \text{Sociology} : n_2 = 14,\ \bar{x}_2 = 23.0,\ s_2 = 12$$

   (a) Find a 95% confidence interval for $\mu_1 - \mu_2$.

   (b) Based upon your answer in (1a), is there a significant difference in the mean time spent socializing between statistics and sociology majors? Why?

   (c) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Based upon your answer in (1a), will H_0 be rejected? Why?

   (d) Based upon your answer in (1c), will the $p-$value be less than 0.05 or greater than 0.05? Why?

   (e) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(f) Based upon your answer in (1e), is there a significant difference between the average socialization times? Why?

(g) Approximate the $p-$value for the test in (1e) using the $t-$table.

(h) Use the $t-$Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the $p-$value for the test in (1e).

(i) Consider the test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.01$ significance level. Based upon your answer in (1g) and (1h), do you reject H_0? Why?

(j) Could we do the above analysis if the study times were *not* normally distributed? Why?

2. The level of iodine in Company A's table salt follows a $N(\mu_1, \sigma_1)$ distribution, while the level in Company B's salt follows a $N(\mu_2, \sigma_2)$ distribution. A random sample of each companies' product yielded

$$\text{Company A} : n_1 = 16,\ \bar{x}_1 = 22.4,\ s_1 = 1.0 \qquad \text{Company B} : n_2 = 9,\ \bar{x}_2 = 27.5,\ s_2 = 1.4$$

Assume that $\sigma_1 = \sigma_2$.

(a) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 < \mu_2$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in (2a), is the mean iodine level for Company A significantly lower than Company B? Why?

(c) Approximate the $p-$value for the test in (2a) using the $t-$table.

(d) Based upon your answer in (2c), is the mean iodine level for Company A significantly lower than Company B? Why?

3. The height of Iowa corn stalks X_1 (in cm) can be modeled by a $X_1 \sim N(\mu_1, \sigma_1)$ distribution, while the height of Nebraska corn stalks X_2 can be modeled by a $X_2 \sim N(\mu_2, \sigma_2)$ distribution. A random sample of corn stalks from Iowa and Nebraska yielded the following summary statistics.

$$\text{Iowa} : n_1 = 15,\ \bar{x}_1 = 155,\ s_1 = 16 \qquad \text{Nebraska} : n_2 = 20,\ \bar{x}_2 = 145,\ s_2 = 10$$

It is known that $\sigma_1 \neq \sigma_2$.

(a) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.10$ significance level. Find the $p-$value for this test using the $t-$table.

(b) Based upon your answer in (3a), is there a significant difference in the mean heights? Why?

(c) Use the $t-$Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the $p-$value for the test in (3a).

(d) Find a 90% confidence interval for $\mu_1 - \mu_2$.

(e) Based upon your answer in (3d), is there a significant difference in the mean heights? Why?

(f) Could we do the above analysis if the heights were *not* normally distributed? Why?

4. The mathematics SAT scores of students at *public* universities have a $N(\mu_1, \sigma_1)$ distribution, while the mathematics SAT scores of students at *private* universities have a $N(\mu_2, \sigma_2)$ distribution. A random sample of students from public and private universities found

$$\text{Public}: n_1 = 16,\ \bar{x}_1 = 520,\ s_1 = 100 \qquad \text{Private}: n_2 = 15,\ \bar{x}_2 = 505,\ s_2 = 85$$

It is known that $\sigma_1 \neq \sigma_2$.

(a) Test $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 > \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the $p-$value for the test in (4a) using the $t-$table.

(c) Based upon your answer in (4b), do students at public universities have a significantly higher mean SAT score? Why?

(d) Use the $t-$Probability Applet at

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http://www.stat.uiowa.edu/~mbognar/applets/t.html
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to precisely determine the $p-$value for the test in (4b).

5. The time (in seconds) it took four individuals to run 1/4 of a mile was recorded. The same individuals went through an exercise program and were re-timed. The times before the exercise program, x_i^b, and after the exercise program, x_i^a, are shown below.

Subject:	1	2	3	4
Before x_i^b:	125	160	185	152
After x_i^a:	127	135	169	115
Difference $x_i^d = x_i^b - x_i^a$:	-2	25	16	37

Assume the difference in running times, X_d, is normally distributed, i.e. $X_d \sim N(\mu_d, \sigma_d)$ (because $n < 30$ this assumption is critical). Because subjects appear in both groups, then groups are *not* independent.

(a) Find the sample mean difference, $\bar{x}_d$.

(b) Verify that the sample standard deviation of the differences $s_d = 16.432$.

(c) We would like to determine if the exercise program significantly *decreases* the mean running time. Thus, test $H_0 : \mu_d = 0$ vs. $H_a : \mu_d > 0$ at the $\alpha = 0.05$ significance level using a paired $t-$test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(d) Approximate the $p-$value for the test in (5c) using the $t-$table.

(e) Based upon your answer in (5d), does the exercise program significantly decrease the mean running time? Why?

(f) Use the $t-$Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the $p-$value for the test in (5d).

(g) Suppose the significance level was $\alpha = 0.10$ (instead of 0.05). Based upon your answer in (5f), does exercise program significantly decrease the mean running time? Why?