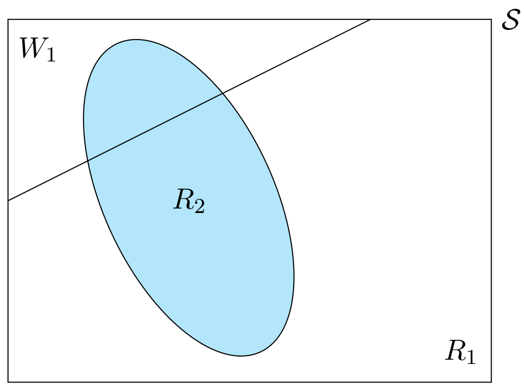

Figure 3.9: Venn diagram showing how selecting a red chip on the first draw (R_1) and selecting a white chip on the first draw (W_1) partition the sample space $\mathcal{S}$; the event of selecting a red chip on the second draw (R_2) is overlaid in blue.

3.6 Exercises

♥ = answers are provided beginning on page 229.

3.1 ♥ Suppose a standard 6-sided die is rolled 4 times. How many outcomes are in the sample space $\mathcal{S}$?

3.2 Suppose two cards are selected from a deck of cards *without* replacement (a deck has 52 cards). How many outcomes are in the sample space $\mathcal{S}$?

3.3 ♥ Suppose a 6-sided die (with sides labeled 1, 2, 3, 4, 5, 6) is rolled 2 times.

(a) Write out the sample space $\mathcal{S}$. *Note that all outcomes are equally likely.*

(b) Let A denote the event that a 1 is obtained on the first roll, and let B denote the event that an even is obtained on the second roll. Find $P(A \text{ and } B)$.

(c) Find the probability that the second roll is exactly twice the first roll.

(d) Find the probability that the second roll is greater than or equal to the first roll.

3.4 Repeat question (3.3) when rolling a 4-sided die.

3.5 ♥ Suppose you randomly select 2 chips *with* replacement from a bowl containing 3 red (R) and 5 white (W) chips. Let R_1 denote the event that a red chip is obtained on the first draw, let R_2 denote the event that a red chip is obtained on the second draw. Find $P(R_1 \text{ and } R_2)$.

3.6 Suppose a die is repeatedly rolled. Find the probability that a 1 is obtained for the first time on the 10th roll. *Hint: To get a 1 for the first time on the 10th roll, you must get a non-one on the 1st roll* (1_1^c)*, a non-one on the second roll* (1_2^c)*,..., a non-one on the 9th roll* (1_9^c)*, and a one on the 10th roll* (1_{10})*.*

3.7 Based on long-run relative frequencies, approximately 51% of all births in the U.S. are boys. Assume independence.

(a) If a woman has 3 children, find the probability that she has all boys.

(b) If a woman has 3 children, find the probability that she does not have all boys.

(c) If a woman has 3 children, find the probability that the first child is a boy, while the last 2 children are girls.

3.8 In reference to question (3.7), answer the following.

(a) If a woman has 3 children, find the probability that she has exactly 1 boy.

(b) If a woman has 3 children, find the probability that she has 1 or more boys.

3.9 ♥ Suppose that 20% of UI students smoke (S), while 30% drink alcohol (A). In addition, 15% smoke *and* drink alcohol.

(a) Given that a student drinks alcohol (A), determine the probability that he/she smokes (S), i.e. find $P(S|A)$.

(b) Are alcohol use and smoking independent? Why?

3.10 It is known that 72% of adults suffer from vision problems. It is also known that 65% of adults suffer from vision problems *and* wear corrective lenses (i.e. eye glasses, contacts). Given that a randomly selected adult suffers from vision problems, find the probability that he/she wears corrective lenses.

3.11 ♥ Suppose a die is rolled one time. Let

$$A = \text{roll a 1} \qquad B = \text{roll an even}$$

(a) Are A and B are mutually exclusive? Why?

(b) Are A and B are independent? Why?

3.12 ♥ Suppose that 80% of computers use an Intel processor.

(a) Suppose 2 computers are randomly selected (*assume independence*). Find the probability that the first computer *or* second computer has an Intel processor, i.e. find $P(I_1 \text{ or } I_2)$.

(b) Suppose 2 computers are randomly selected (*assume independence*). Find the probability that exactly 1 has an Intel processor.

(c) Suppose computers are repeatedly selected at random (*assume independence*). Find the probability that the 10th computer is the first without an Intel processor.

(d) Suppose 4 computers are randomly selected (*assume independence*). Find the probability that 1 or more contain an Intel processor.

3.13 Suppose that 4% of desktop computers run the Linux operating system (L). Suppose 2 computers are randomly selected (*assume independence*).

(a) Find the probability that neither computer is running Linux.

(b) Find the probability that the first computer runs Linux (L_1) *or* the second computer runs linux (L_2).

(c) Find the probability that exactly one of the computers runs Linux.

(d) Suppose computers are repeatedly selected (assume independence). Find the probability that the 4th selected computer is the first one running Linux.

3.14 A slot machine has 3 wheels. Each wheel has 10 symbols, and each symbol is equally likely when the wheel is spun (assume the wheels act independently of each other). The middle wheel has 1 bell among its 10 symbols, while the left and right wheels have 4 bells each.

(a) You win the jackpot if the wheels show all bells. What is the probability of winning the jackpot on any given spin?

(b) Find the probability of obtaining exactly 2 bells on any given spin.

(c) What is the probability that you get a bell on the first wheel *or* third wheel?

3.15 Worldwide, approximately 70% of smart phones run the Android operating system (A) while 30% run a non-Android operating system (A^c). Suppose 2 smart phones are randomly selected (assume independence).

(a) Determine the probability that the first runs Android (A_1) *or* the second runs Android (A_2).

(b) Determine the probability that exactly 1 runs Android.

3.16 ♥ Suppose a bowl has 8 chips; 4 of the chips are black (B), the remaining 4 chips are red (R). One of the black chips is labeled 1, while the other black chips are labeled 2, 3, and 4 respectively. The red chips are labeled 2, 4, 5, and 7 respectively. Hence, the chips in the bowl could be labeled as

$$B1, B2, B3, B4, R2, R4, R5, R7$$

(a) Suppose one chip is randomly selected. Let A be the event that an even is obtained, and B be the event that a black is obtained. Are A and B independent? Why?

(b) Suppose two chips are randomly selected *without* replacement. Let A be the event that a 7 is obtained on the *first* draw, and B be the event that a red is obtained on the *second* draw. What is $P(B|A)$?

(c) Suppose two chips are randomly selected *without* replacement. Let A be the event that a 7 is obtained on the *first* draw, and B be the event that a red is obtained on the *second* draw. Find $P(A \text{ and } B)$.

(d) Suppose 1 chip is randomly selected from the bowl. Let A be the event that an even is obtained, and B be the event that a black is obtained. Are A and B mutually exclusive? Why? What is $P(A \text{ or } B)$?

3.17 Suppose a box contains 12 silver coins (S) and 3 gold coins (G).

(a) If you randomly select 2 coins *without* replacement, determine the probability that the first coin is silver (S_1) *and* the second coin is gold (G_2).

(b) Use the complement rule to find the probability that 1 or fewer gold coins are selected.

3.18 In reference to question (3.17), answer the following.

(a) If you randomly select 2 coins *without* replacement, determine the probability that you obtain exactly 1 gold coin (G).

(b) If you randomly select 3 coins *without* replacement, determine the probability that you obtain exactly 1 gold coin (G).

3.19 Suppose a die is rolled. Consider the following events:

$$\begin{aligned} A &= 2,4 \text{ or } 6 \text{ is rolled} \\ B &= 1,2 \text{ or } 5 \text{ is rolled} \\ C &= 3 \text{ or } 5 \text{ is rolled} \end{aligned}$$

(a) Are A and B are mutually exclusive? Why?

(b) Are A and C are mutually exclusive? Why?

(c) Find $P(A|B)$

(d) Find $P(B \text{ or } C)$.

(e) Are A and B independent? Why?

(f) Are B and C independent? Why?

3.20 Suppose A and B are independent where $P(A) = 0.3$ and $P(B|A) = 0.5$. Find $P(A \text{ or } B)$.

3.21 ♥ Let A and B be two independent events. Suppose $P(A \text{ or } B) = 0.6$ and $P(A|B) = 0.2$. What is $P(B)$?

3.22 Suppose $P(A) = 0.6$, $P(B|A) = 0.1$, and $P(B|A^c) = 0.3$. What is $P(B)$?

3.23 Suppose events A and B are mutually exclusive where $P(A) = 0.5$ and $P(B) = 0.2$. What is $P(A|B)$?

3.24 ♥ Let A and B be two events. Suppose $P(A) = 0.8$, $P(B|A) = 0.1$, and $P(A \text{ or } B) = 0.9$. What is $P(B)$?

3.25 Let A and B be two events. Suppose $P(A) = 0.5$, $P(B) = 0.8$, and $P(A \text{ or } B) = 0.9$. What is $P(B|A)$?

3.26 Suppose you roll a standard 6-sided die. If you roll a "1" (1), you randomly select one chip from a bowl containing 2 red (R) and 3 white (W) chips. If you don't roll a "1" (1^c), you randomly select 1 chip from a bowl containing 7 red (R) and 3 white (W) chips.

(a) Find the probability that you roll a "1" *and* obtain a white chip (W).

(b) Determine the probability you obtain a white chip, i.e. find $P(W)$.

(c) Given that a white chip was obtained, determine the probability that a 1 was rolled on the die, i.e. find $P(1|W)$.

(d) Determine the probability that a 1 is rolled on the die *or* a red chip is selected from the bowl, i.e. find $P(1 \text{ or } R)$.

3.27 A technician is assigned the task of examining transistors before they are installed into a radio. She has a box containing 12 transistors, 3 of which are defective.

(a) Suppose 2 transistors are randomly selected *with* replacement. Find the probability that both are defective (i.e. find $P(D_1 \text{ and } D_2)$). Assume independence.

(b) Suppose 2 transistors are randomly selected *with* replacement. Find the probability that the first is defective *or* the second is defective, i.e. find $P(D_1 \text{ or } D_2)$.

(c) Suppose 2 transistors are randomly selected *without* replacement. Given that the first transistor is defective, determine the probability that the second transistor is defective (i.e. find $P(D_2|D_1)$).

(d) Suppose 2 transistors are randomly selected *without* replacement. Find the probability that both are defective (i.e. find $P(D_1 \text{ and } D_2)$).

(e) Suppose 2 transistors are randomly selected *without* replacement. Find the probability that the first is defective *or* the second is defective, i.e. find $P(D_1 \text{ or } D_2)$. *Hint: Use the law of total probability to find* $P(D_2)$.

3.28 There are 52 cards in a deck of cards, 13 of which are hearts. Suppose you randomly select 2 cards *without* replacement. Let H_1 denote the event that a heart is obtained on the first draw, and let H_2 denote the event that a heart is obtained on the second draw.

(a) Use the law of total probability to find the probability you obtain a heart on the second draw, i.e. find $P(H_2)$.

(b) Find the probability both cards are hearts, i.e. find $P(H_1 \text{ and } H_2)$.

(c) Find the probability that a heart is obtained on the first draw *or* second draw, i.e. find $P(H_1 \text{ or } H_2)$.

(d) What is the probability that a heart was obtained on the first draw given that a heart is obtained on the second draw? In other words, find $P(H_1|H_2)$.

3.29 ♥ It is known that 8% of people in the U.S. use illegal drugs (i.e. $P(D) = 0.08$). Given that a person is using illegal drugs, a drug test will (correctly) return a positive result with probability 0.97 (i.e. $P(+|D) = 0.97$). The specificity of the drug test is 0.99.

(a) Find the probability that a randomly selected person will test positive for illegal drugs (i.e. find $P(+)$).

(b) Given that a randomly selected person tested *positive*, what is the probability that he/she uses illegal drugs?

(c) Determine the probability of a false negative test result.

(d) Find the probability that a randomly chosen person takes illegal drugs *or* tests positive (i.e. find $P(D \text{ or } +)$).

3.30 The probability that a passenger will attempt to board an airplane with illegal drugs is 0.005 (i.e. $P(D) = 0.005$). Given that a passenger has illegal drugs, the probability that the alarm will sound is 0.97 (i.e. $P(A|D) = 0.97$). If a passenger does not have illegal drugs, the probability that the alarm will not sound is 0.95 (i.e. $P(A^c|D^c) = 0.95$).

(a) What is the sensitivity of the drug detection machine?

(b) What is the specificity of the drug detection machine?

(c) Find the probability that the alarm does not sound given that the passenger is carrying drugs (i.e. find $P(A^c|D)$).

(d) Suppose a passenger is randomly selected. Find the probability that the alarm sounds when he/she enters security (i.e. find $P(A)$).

(e) Given that the alarm sounds, find the probability that the passenger actually has illegal drugs. (i.e. find $P(D|A)$). This quantity is known as the "predictive value of a positive test".

(f) Find the "predictive value of a negative test" (i.e. find $P(D^c|A^c)$). In words, what does this quantity mean?

3.31 A farm has two types of trees: 30% are orange trees (O) and 70% are apple trees (A). Frost (F) has damaged 40% of the orange trees (i.e. $P(F|O) = 0.40$) and 10% of the apple trees.

(a) Find the probability that a randomly selected tree was damaged by frost *and* is an apple tree.

(b) Find the probability that a randomly selected tree has been damaged by frost.

(c) Given that a randomly selected tree has been damaged by frost, determine the probability that it is an apple tree.

3.32 ♥ An egg manufacturer produces 3 sizes of eggs: 40% are classified as *small*, 50% are *large*, and 10% are *extra-large*. It is known that 8% of the small, 10% of the large, and 14% of the extra-large eggs are cracked.

(a) If an egg is randomly chosen, find the probability that it is cracked.

(b) Given that a randomly chosen egg is cracked, find the probability that it is an *extra-large* egg.

(c) If you randomly select 3 eggs, determine the probability that exactly 1 is cracked.

3.33 A chest has 3 drawers: the first drawer (D_1) has 2 silver coins (S), the second drawer (D_2) has 1 silver (S) and 1 gold (G) coin, and the thrid drawer (D_3) has 2 gold (G) coins. A drawer is randomly selected and a coin is randomly selected from the chosen drawer. Lets refer to the selected coin as coin 1, and the other coin in the drawer is coin 2.

(a) Use the law of total probability to determine the probability that the first coin selected from the drawer is gold, i.e. find $P(G_1)$.

(b) Find the probability that both coins in the selected drawer are gold, i.e. find $P(G_1 \text{ and } G_2)$. *This one is easy. Both coins are gold only if we are selecting from* D_3*, hence* $P(G_1 \text{ and } G_2) = P(D_3)$.

(c) Given that the selected coin is gold, determine the probability that other coin in the drawer is also gold, i.e. find $P(G_2|G_1)$. Briefly interpret your result.