The p−value is found in the usual way:

$$\begin{aligned} p-\text{value} &= P(t_{(n-1)} < t^*) \\ &= P(t_{(5)} < -3.22) \\ &\in (0.01, 0.02) \end{aligned}$$

Since the p−value is less than α, reject H_0. Benadryl significantly increases the mean reaction time. A 95% (two-sided) confidence interval for μ_d can be found in the expected way:

$$\begin{aligned} \bar{x}_d \pm t_{\alpha/2,n-1}\frac{s_d}{\sqrt{n}} &= -0.1267 \pm 2.571\frac{0.0963}{\sqrt{6}} \\ &= -0.1267 \pm 0.101 \\ &= (-0.228, -0.026) \end{aligned}$$

The confidence interval excludes 0, thus Benadryl significantly affects the mean reaction time (in fact, since the confidence interval lies entirely below 0, then Benadryl significantly *increases* the mean reaction time). Note: Because our sample size is 6 (i.e. $n < 30$) then the normal assumption is critical. If $n \geq 30$, then this normality assumption can be relaxed (simply replace the t−distribution with a Z).

8.4 Exercises

♥ = answers are provided beginning on page 229.

8.1 ♥ A sociologist collected a random sample of 13 statistics majors and 14 sociology majors. The students were asked about how many hours per week they spend socializing. The results are summarized in the following table. Assume that the amount of socialization for statistics majors follows a normal distribution with mean μ_1 and standard deviation σ_1, while the amount of socialization for sociology majors follows a normal distribution with mean μ_2 and standard deviation σ_2 . Because the sample standard deviations s_1 and s_2 are quite similar, lets make the reasonable assumption that $\sigma_1 = \sigma_2$.

Statistics:	$n_1 = 13$	$\bar{x}_1 = 32.1$	$s_1 = 10$
Sociology:	$n_2 = 14$	$\bar{x}_2 = 23.0$	$s_2 = 12$

(a) Find a 95% confidence interval for $\mu_1 - \mu_2$.

(b) Based upon your answer in 8.1(a), is there a significant difference in the mean time spent socializing between statistics and sociology majors? Why?

(c) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Based upon your answer in 8.1(a), will H_0 be rejected? Why?

(d) Based upon your answer in 8.1(c), will the p−value be less than 0.05 or greater than 0.05? Why?

(e) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(f) Based upon your answer in 8.1(e), is there a significant difference between the average socialization times? Why?

(g) Approximate the p−value for the test in 8.1(e) using the t−table.

(h) Use the t−Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p−value for the test in 8.1(e).

(i) Consider the test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.01$ significance level. Based upon your answer in 8.1(g) and 8.1(h), do you reject H_0? Why?

8.2 The level of iodine in Company A's table salt follows a $N(\mu_1, \sigma_1)$ distribution, while the level in Company B's salt follows a $N(\mu_2, \sigma_2)$ distribution. A random sample of each companies' product yielded

Company A:	$n_1 = 16$	$\bar{x}_1 = 22.4$	$s_1 = 1.0$
Company B:	$n_2 = 9$	$\bar{x}_2 = 27.5$	$s_2 = 1.4$

Assume that $\sigma_1 = \sigma_2$.

(a) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 < \mu_2$ at the $\alpha = 0.01$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in 8.2(a), is the mean iodine level for Company A significantly lower than Company B? Why?

(c) Approximate the p−value for the test in 8.2(a) using the t−table.

(d) Based upon your answer in 8.2(c), is the mean iodine level for Company A significantly lower than Company B? Why?

8.3 A researcher recently examined the studying habits of college students. It is known that the number of hours per week spent studying by Communications majors follows a $N(\mu_1, \sigma_1)$ distribution. Also, it is known that the number of hours per week spent studying by English majors follows a $N(\mu_2, \sigma_2)$ distribution. A random sample of 16 Communications majors and 26 English majors found:

Communications:	$n_1 = 16$	$\bar{x}_1 = 5.5$	$s_1 = 2.0$
English:	$n_2 = 26$	$\bar{x}_2 = 7.3$	$s_2 = 2.4$

Assume the population standard deviations are equal, i.e. $\sigma_1 = \sigma_2$.

(a) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.01$ significance level. Approximate the p−value for this test using the t−table.

(b) (*3 pts*) Based upon your answer in 8.3(a), is there a significant difference in the population mean study times? Why?

(c) Use the t−Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p−value for the test in 8.3(a).

(d) Find a 99% confidence interval for $\mu_1 - \mu_2$.

(e) (*3 pts*) Based upon your answer in 8.3(d), is there a significant difference in the population mean study times? Why?

8.4 ♥ The weekly time (in hours) spent using a cell phone for UI females follows a $N(\mu_1, \sigma_1)$ distribution, while weekly phone times for UI males follows a $N(\mu_2, \sigma_2)$ distribution. A random sample of each gender yielded

Female:	$n_1 = 18$	$\bar{x}_1 = 22$	$s_1 = 8$
Male:	$n_2 = 20$	$\bar{x}_2 = 17$	$s_2 = 4$

Assume $\sigma_1 \neq \sigma_2$.

(a) Find a 95% confidence interval for $\mu_1 - \mu_2$.

(b) Based upon your answer in 8.4(a), is there a significant difference in phone usage? Why?

(c) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Find the p−value for this test using the t−table.

(d) Based upon your answer in 8.4(c), is there a significant difference in mean phone usage? Why?

(e) Use the t−Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p−value for the test in 8.4(c).

(f) Suppose $\alpha = 0.01$ instead of 0.05. Based upon your answer in 8.4(c), is there a significant difference in phone usage? Why?

8.5 The height of Iowa corn stalks (in cm) have a $N(\mu_1, \sigma_1)$ distribution, while the height of Nebraska corn stalks have a $N(\mu_2, \sigma_2)$ distribution. A random sample of corn stalks from Iowa and Nebraska yielded the following summary statistics.

Iowa:	$n_1 = 15$	$\bar{x}_1 = 155$	$s_1 = 16$
Nebraska:	$n_2 = 20$	$\bar{x}_2 = 145$	$s_2 = 10$

It is known that $\sigma_1 \neq \sigma_2$.

(a) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.10$ significance level. Find the p−value for this test using the t−table.

(b) Based upon your answer in 8.5(a), is there a significant difference in the mean heights? Why?

(c) Use the t−Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p−value for the test in 8.5(a).

(d) Find a 90% confidence interval for $\mu_1 - \mu_2$.

(e) Based upon your answer in 8.5(d), is there a significant difference in the mean heights? Why?

8.6 The mathematics SAT scores of students at *public* universities have a $N(\mu_1, \sigma_1)$ distribution, while the mathematics SAT scores of students at *private* universities have a $N(\mu_2, \sigma_2)$ distribution. A random sample of students from public and private universities found

Public:	$n_1 = 16$	$\bar{x}_1 = 520$	$s_1 = 100$
Private:	$n_2 = 15$	$\bar{x}_2 = 505$	$s_2 = 85$

It is known that $\sigma_1 \neq \sigma_2$.

(a) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 > \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Based upon your answer in 8.6(a), do students at public universities have a significantly higher mean SAT score? Why?

(c) Find the p–value for the test in 8.6(a) using the t–table.

(d) Based upon your answer in 8.6(c), do students at public universities have a significantly higher mean SAT score? Why?

(e) Use the t–Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p–value for the test in 8.6(c).

8.7 ♥ The hourly wages of male fast food workers have a $N(\mu_1, \sigma_1)$ distribution, while the hourly wages of female fast food workers have a $N(\mu_2, \sigma_2)$ distribution. A random sample of 17 male and 13 female workers found:

Male:	$n_1 = 17$	$\bar{x}_1 = \$6.60$	$s_1 = \$1.25$
Female:	$n_2 = 13$	$\bar{x}_2 = \$8.40$	$s_2 = \$2.95$

It is known that $\sigma_1 \neq \sigma_2$. *This exercise demonstrates that a one-sided test has more statistical power than a two-sided test.*

(a) Test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 < \mu_2$ at the $\alpha = 0.05$ significance level. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(b) Find the p–value for the test in 8.7(a) using the t–table.

(c) Based upon your answer in 8.7(b), do male workers make significantly less than female workers, on average? Why?

(d) Use the t–Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p–value for the test in 8.7(b).

(e) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Find the p–value for this test using the t–table.

(f) Based upon your answer in 8.7(e), is there a significant difference in the mean wages? Why?

(g) Verify that the 95% (two-sided) confidence interval for $\mu_1 - \mu_2$ contains 0.

8.8 The time (in seconds) it took four individuals to run 1/4 of a mile was recorded. The same individuals went through an exercise program and were re-timed. The times before the exercise program, x_i^b, and after the exercise program, x_i^a, are shown below.

Subject:	1	2	3	4
Before x_i^b:	125	160	185	152
After x_i^a:	127	135	169	115
Difference $x_i^d = x_i^b - x_i^a$:	−2	25	16	37

Assume the difference in running times, X_d, is normally distributed, i.e. $X_d \sim N(\mu_d, \sigma_d)$ (because $n < 30$ this assumption is critical). Because subjects appear in both groups, then groups are *not* independent.

(a) Find the sample mean difference, $\bar{x}_d$.

(b) Verify that the sample standard deviation of the differences $s_d = 16.432$.

(c) We would like to determine if the exercise program significantly *decreases* the mean running time. Thus, test

$$H_0 : \mu_d = 0$$
$$H_a : \mu_d > 0$$

at the $\alpha = 0.05$ significance level using a paired t−test. *Find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion.*

(d) Approximate the p−value for the test in 8.8(c) using the t−table.

(e) Based upon your answer in 8.8(d), does the exercise program significantly decrease the mean running time? Why?

(f) Use the t−Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the p−value for the test in 8.8(d).

(g) Suppose the significance level was $\alpha = 0.10$ (instead of 0.05). Based upon your answer in 8.8(f), does exercise program significantly decrease the mean running time? Why?