9.1 Hypothesis Testing

Introduction to Hypothesis Testing

Null hypothesis (H_0)?

Or the Alternative hypothesis (H_a)?

Recall: Confidence Interval for p

- 95% Confidence Interval for p

$$\hat{p} - 2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}} \quad \text{to} \quad \hat{p} + 2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$$

95% Confidence Interval (CI)

- Provides plausible values for the unknown population proportion, p.
- We have confidence in the process that produced this interval.
- We are 95% confident that the true population proportion p lies in the CI.

95% Confidence Interval

- General idea:
  - The population proportion, p, could be any of the values in the interval.
  - Values outside the interval are **not plausible** values for p.

- If someone claims a true value for p that is outside of the CI, we should be skeptical of their claim.

95% Confidence Interval

- We can use this idea of plausible vs. non-plausible values provided by a CI to make important decisions.
- After we formulate a hypothesis (or research question), we can then collect evidence (data) to make a decision on our hypothesis.

Example

- A university has found in the past that out of all students who are offered admission, the proportion who accept is 0.70. After a new director of admissions is hired, the group of concerned faculty wants to check if the proportion of students accepting has changed significantly.

- How should the faculty make its decision?

Example

- Population of interest:
  - All students who have been offered admission after the director was hired.
- Parameter:
  - Proportion p of this population that accepts admission.

Example

Form the set of hypotheses (related to research question)

- Null Hypothesis
  - H_0: $p = 0.70$ ← Director hasn't changed things
- Alternative Hypothesis
  - H_A: $p \neq 0.70$ ← Director has changed things (could be for the better or worse)

Example

Form the hypotheses

- Null Hypothesis
  - H_0: $p = 0.70$
- Alternative Hypothesis
  - H_A: $p \neq 0.70$

Usually, the 'more interesting' outcome is placed in the alternative hypothesis. Here, it's the hypothesis that could lead to rewarding or demoting the director.

Example

Form the hypotheses

- Null Hypothesis
  - H_0: $p = 0.70$
- Alternative Hypothesis
  - H_A: $p \neq 0.70$

HYPOTHESIS TESTING PROCEDURE:

We will assume the null hypothesis to be true… akin to INNOCENT UNTIL PROVEN GUILTY.

Then we will gather evidence (data) to test this hypothesis.

If we have overwhelming evidence against the null hypothesis (here, that $p \neq 0.70$), we reject the null in favor of the alternative.

Example

n=1000

- The faculty group contacts 1000 students at random who have been offered admission and finds out that 750 of them have accepted.

$$\hat{p} = \frac{750}{1000} = 0.75$$

- Is this sufficient evidence for the university to conclude that the acceptance rate has changed?

Example

- 95% confidence interval for p

$$\hat{p} - 2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}} \quad \text{to} \quad \hat{p} + 2\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$$

$$0.75 - 2\sqrt{\frac{0.75\left(0.25\right)}{1000}} \quad \text{to} \quad 0.75 + 2\sqrt{\frac{0.75(0.25)}{1000}}$$

$$0.75 \pm 0.03$$

Example

- 95% confidence interval for p.

$$(0.72, 0.78)$$

- Based on the collected data, the plausible values for the acceptance rate lie within this interval.

- Is there evidence that the proportion is NOT 0.70, but rather, something else?

Example $(0.72, 0.78)$

- Based on our 95% CI, there IS evidence that the proportion is NOT 0.70, but rather, something larger.

- Null Hypothesis
  - H_0: $p = 0.70$

We will reject H_0 in this case,

- Alternative Hypothesis
  - H_A: $p \neq 0.70$

in favor of H_A

How do confidence intervals relate to Hypothesis Testing?

- One way to test a hypothesis about a parameter is to compute a confidence interval.
- All hypothesis tests are performed at a specific **significance level** (or α-level).
- When we use a CI to perform a hypothesis test, we must make sure our confidence level coincides with our significance level…

How do confidence intervals relate to Hypothesis Testing?

- We use a 95% confidence interval to perform a hypothesis test at the $\alpha=0.05$ significance level.

- We use a 90% confidence interval to perform a hypothesis test at the $\alpha=0.10$ significance level.

- We focused on 95% confidence intervals in this class, but you can actually choose the level of confidence you want (90%, 95%, etc.).

Some Definitions

A **hypothesis** is a claim about a population parameter, such as a population proportion (p) or population mean (μ).

A **hypothesis test** is a standard procedure for testing a claim about a population parameter.

Hypotheses come in pairs

- There are always at least two hypotheses in any hypothesis test.
- In the admissions example, the two hypotheses are essentially:
  - $H_{0:}$ That the acceptance rate is 0.70 (**null**)
  - H_a: That the acceptance rate is something other than 0.70 (**alternative**)

Null and Alternative Hypotheses

The **null hypothesis**, or H_0, is the starting assumption for a hypothesis test. For the types of hypothesis tests in this chapter, the null hypothesis always claims a specific value for a population parameter and therefore takes the form of an equality:

H_0: population parameter = claimed value

The **alternative hypothesis**, or H_a, is a claim that the population parameter has a value that differs from the value claimed in the null hypothesis. It may take one of the following forms:

(left-tailed) H_a: population parameter < claimed value

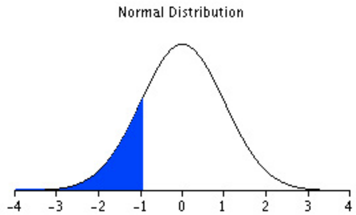
(right-tailed) H_a: population parameter > claimed value

(two-tailed) H_a: population parameter ≠ claimed value

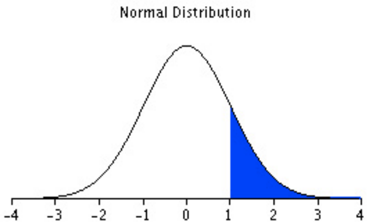
The Alternative Hypothesis

(only one of the three applies for each research question)

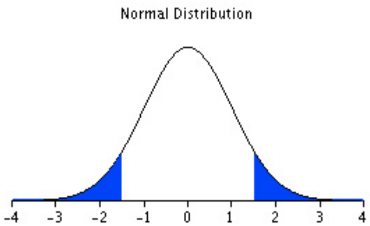
- Left-tailed hypothesis test
  - "less than"
  - Population parameter < claimed value



- Right-tailed hypothesis test
  - "greater than"
  - Population parameter > claimed value



- Two-tailed hypothesis test
  - "Not equal to"
  - Population parameter ≠ claimed value



Two Possible Outcomes of a Hypothesis Test

1. ***Reject* the null** hypothesis, H_0, in which case we have evidence in support of the alternative hypothesis.

2. ***Do not reject* the null** hypothesis, H_0, in which case we do not have enough evidence to support the alternative hypothesis.